Graph Abstractions in Wireless Networking

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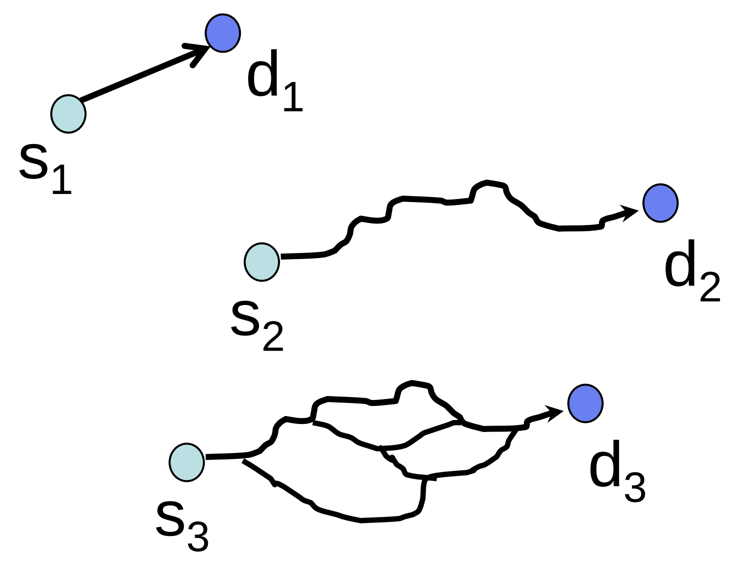
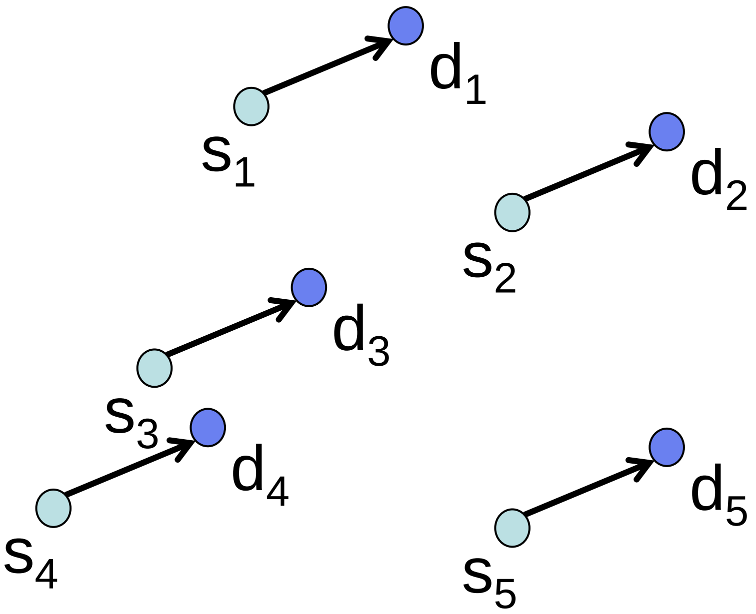
Utilizing wireless networks

- *Given*: Sources, destinations, and demands.
- *Optimize*: „Throughput“
- *Using*:
  - Channels (frequencies, codes)
  - Power control
  - Time multiplexing
  - Space diversity
  - Routing
  - Bit-rate adjustment



Core subproblem: Link Scheduling

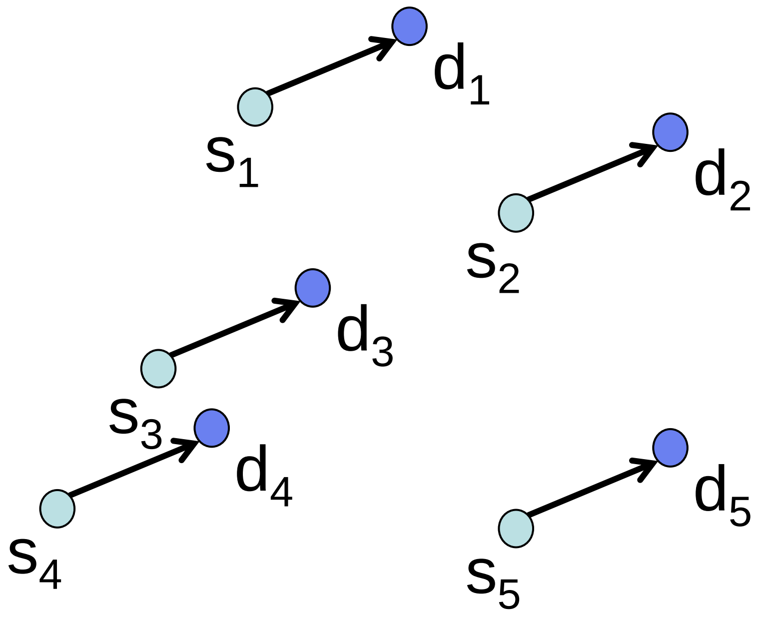
- *(Shortest) Link Scheduling problem*
- *Given*: Links (sources, destinations)
- *Using*:
  - ~~Channels (frequencies, codes)~~
  - Power control
  - (TDMA) scheduling
  - Space diversity
  - ~~Routing~~
  - ~~Rate adjustment~~



- *Minimize the number of slots used*

Related problem: MIS

- *MIS = „Max Independent Set of Links“,*
  - *„One-shot scheduling“*
- *Given*: Links (sources, destinations)
- *Using*:
  - ~~Channels (frequencies, codes)~~
  - Power control
  - ~~(TDMA) scheduling~~
  - Space diversity
  - ~~Routing~~
  - ~~Rate adjustment~~



- *Maximize **number** of links in a single slot*

Issues at Heart

Coloring graphs,   with few colors

What‘s the model?

Issues at Heart

Coloring „graphs“, with few colors

What‘s the model?

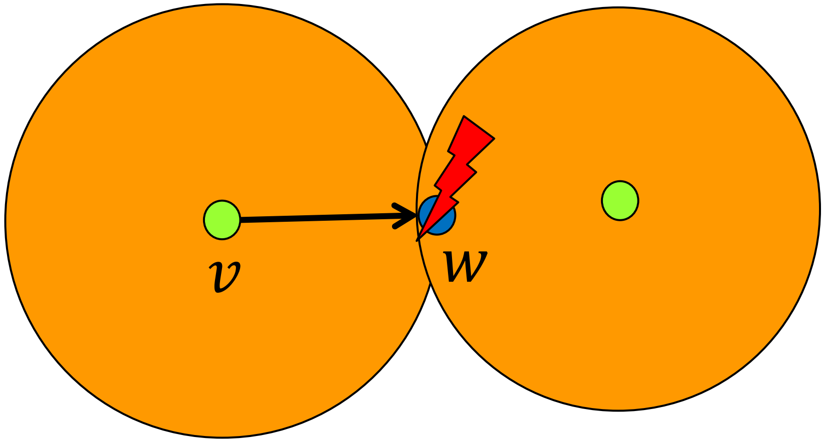
Wireless communication

Node v successfully communicates to node w if:

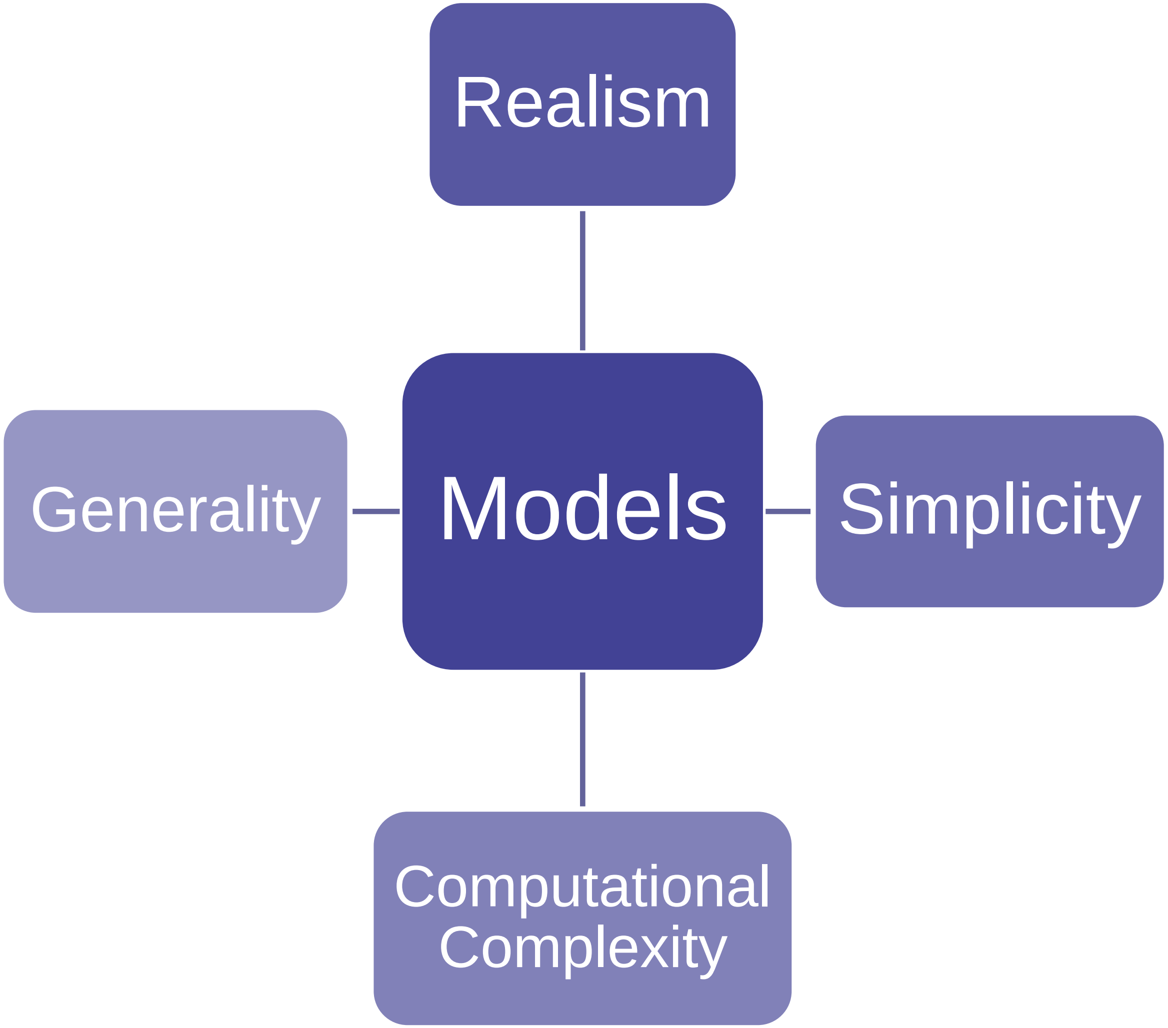
i. v transmits (towards w)

ii. v can communicate with w (~ is a neighbor)

iii. w doesn‘t experience (too much) *interference*



Which problem → Which model?

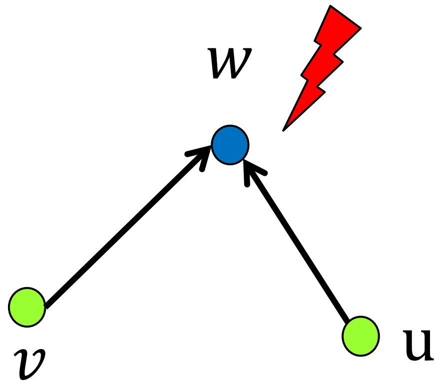


Ground model: Radio networks

- Receiving node experiences interference if:

$$\Leftrightarrow$$

2+ neighbors transmit



Issues with the Radio Networks model

#1 : Computational complexity

Approximate (Graph) Coloring is provably intractable

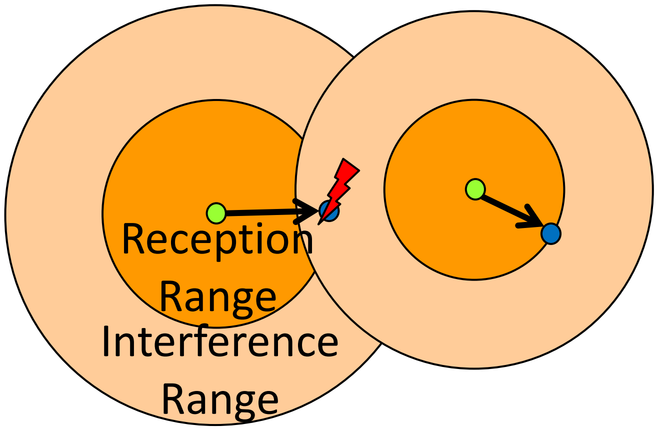
Possible (& necessary) solution: Restrict the graph class

Issues with the Radio Networks model

#2 : Modeling communication and interference with the same graph

Possible solution:

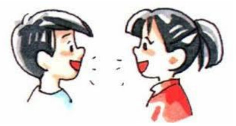
- Introduce a super-graph for interference
- (Again) Restricted graph classes



Issues with the Radio Networks model

#3 : Treating interference as a binary property

Interference adds up



<u>What matters:</u>

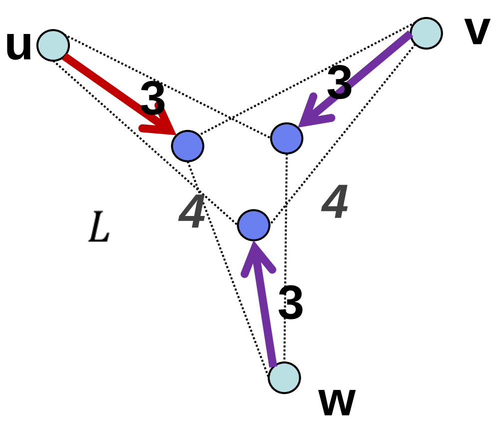
Is the received signal strength sufficiently large compared with the interference+noise?

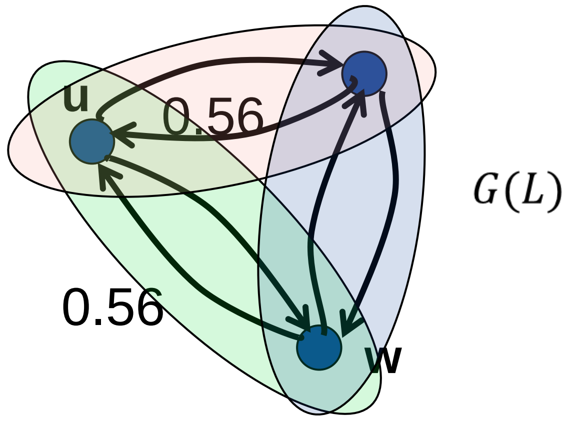
➔ „Feasibility“ is a complicated independence system.

„Physical“ or SINR model

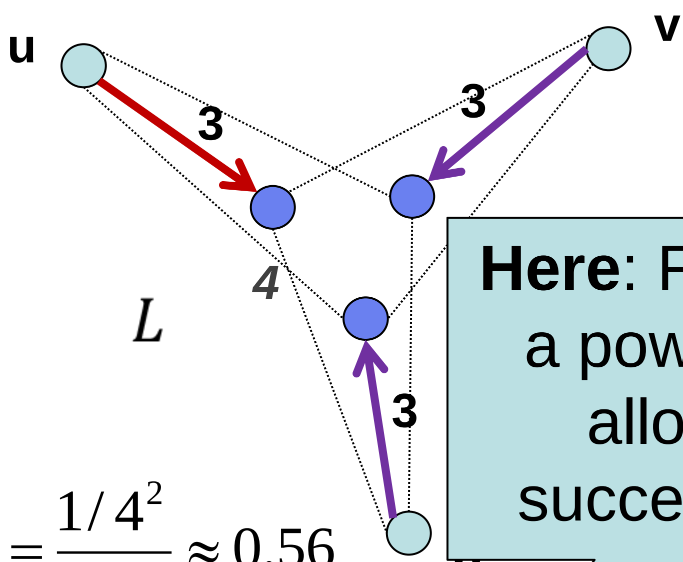
1. Interference is *additive*
2. Signal strength/interference decreases polynomially with distance
3. <u>Affectance</u> (=Relative interference) threshold

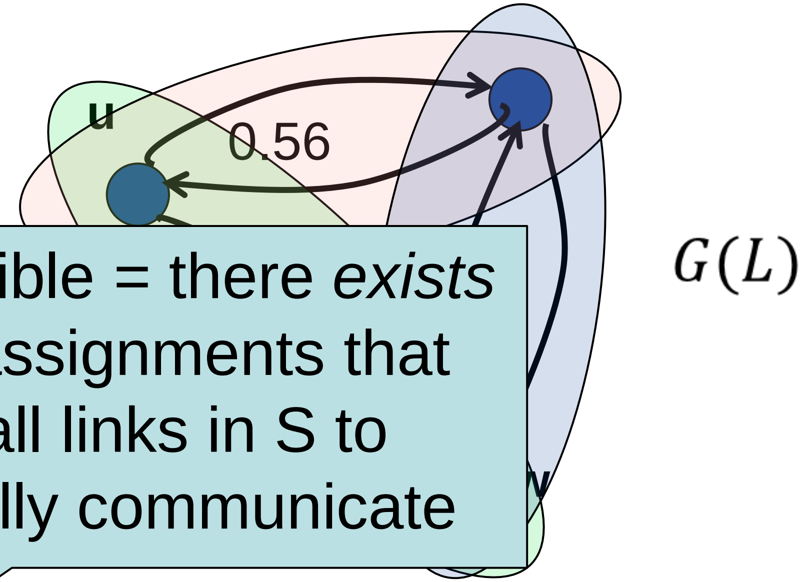
   = Strength of interference / Strength of (intended) signal





Feasibility in the SINR model





Here: Feasible = there *exists* a power assignments that allows all links in S to successfully communicate

$$a_u(w) = \frac{1/4^2}{1/3^2} \approx 0.56$$

Given set L of links ... an edge-weighted digraph $G(L)$.
Weight of edge i ... Relative interference of link i on link j

A set S is ***feasible*** iff
the weighted in-degree of every link within $G(S)$ is $< 1/\beta$

Properties of SINR model

- Not binary
- Not symmetric
- Faraway interference

- The math can get ugly
  - Intuition is hard to come by

Surprises in SINR

- Leader election in $O(\log n)$ rounds
  - [Fineman, Gilbert, Kuhn, Newport, PODC'16]
  - Compares with $\Theta(log^2 n)$ for radio networks
- Leader election in 2 rounds
  - [H, Holzer, Markatou , SIROCCO'17]
  - Requires exponential amount of power control
- Scale-free-ness
  - No fixed radius, or fixed set of interferers
- Power control can be very powerful

Approximation Results on MIS in SINR model

MIS has constant-factor approximations for:

- Uniform power in $\mathbb{R}^2$.
  [Goussevskaia,H,Wattenhofer,Welzl'09]
- Other fixed power in general metrics [H, Mitra, SODA'11]
- Arbitrary power control [Kesselheim, SODA'11]
  - Also, with power limitations [Wan'12, Kesselheim'12]
- Variable bit-rates [Kesselheim'12]
- Uniform power with spectrum sharing [H,Mitra'12]
  - with distributed learning [Asgeirsson, Mitra ,'11]
  - under jamming [Dams et al...]
- Holds also for an extension to Rayleigh fading
  [Dams, Hoefer, Kesselheim '13], [H, '16]

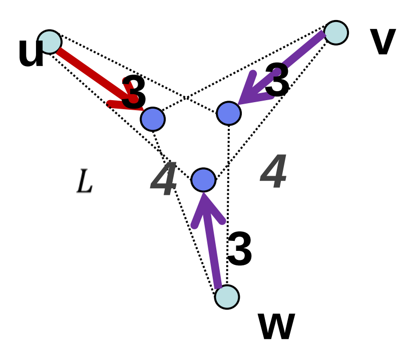
Scheduling in SINR model

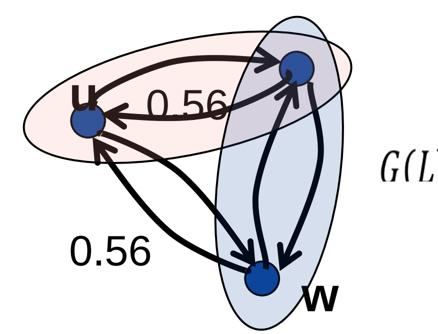
Scheduling approximation:

- $O(\log n)$-approximation [Direct from MIS results]
- $O(\log \Delta)$-approximation (Δ = link length diversity)

- Known algorithms give $\Omega(\log n)$-approximation [HKT '15]
- None of the previous techniques suffice to improve the performance guarantee

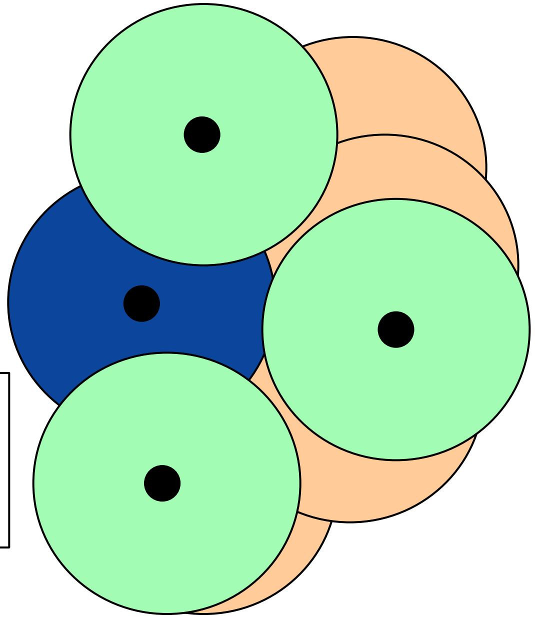
- $\Rightarrow$ Edge-weighted graphs are harder than graphs





Graph models: Unit Disc Graphs

- Nodes = transmitters/receivers in the plane
- Adjacent nodes := distance < 1
- Edge ➔ Communication
- & Interference



UDGs are 3-inductive independent
Disc graphs are 5-inductive independent

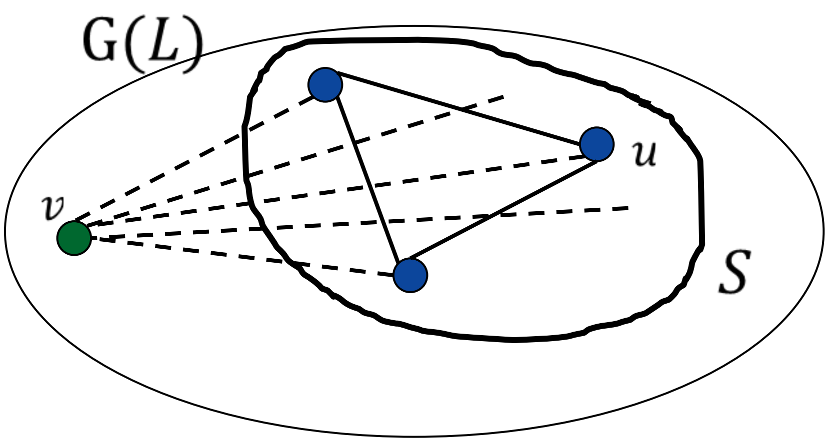
H is k-<u>inductive independent</u> if
every induced subgraph H contains vertex v with $\alpha(N[v]) \leq k$

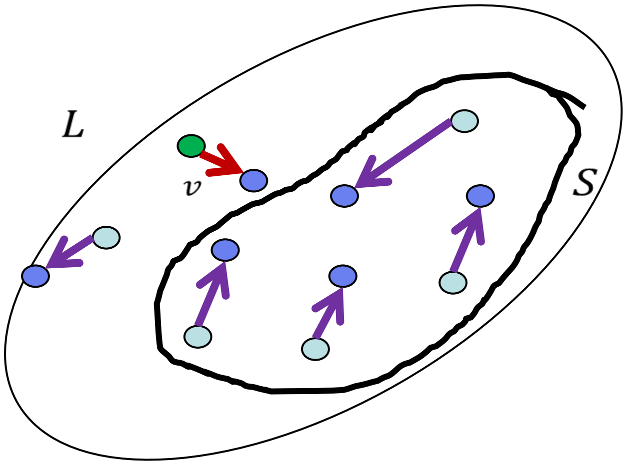
How do these algorithms work?

- The edge-weighted instances induced by wireless links are „sparse“

Kesselheim, SODA‘11,
H, Holzer, Mitra, Wattenhofer, SODA‘13

- The *weighted inductive independence* is constant.
  - Greedy algorithms achieve constant approximation for Capacity

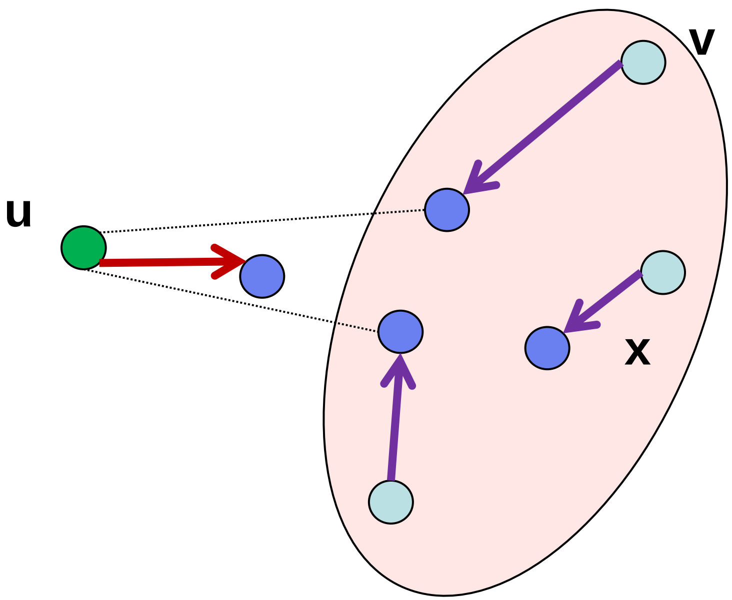


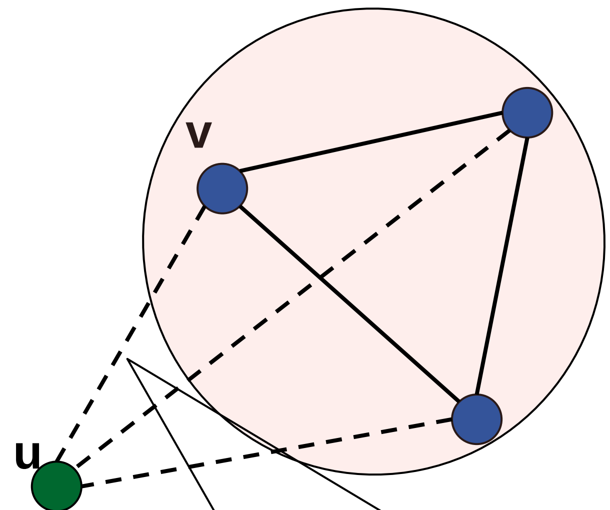


Weighted inductiveness

Kesselheim, Vöcking, DISC'10

- A node u in an edge-weighted graph $G = (V, E, w)$ is **t-good** if $w(u, S) \leq t$, for any feasible set $S \subseteq V$.
- A set of nodes is is ***t-inductive independent*** if any subset contains a t-good node
- $\rho(L) =$ smallest *t* s.t. $G(L)$ is t-inductive independent
- Gives a t-inductive ordering == ordering links by increasing length



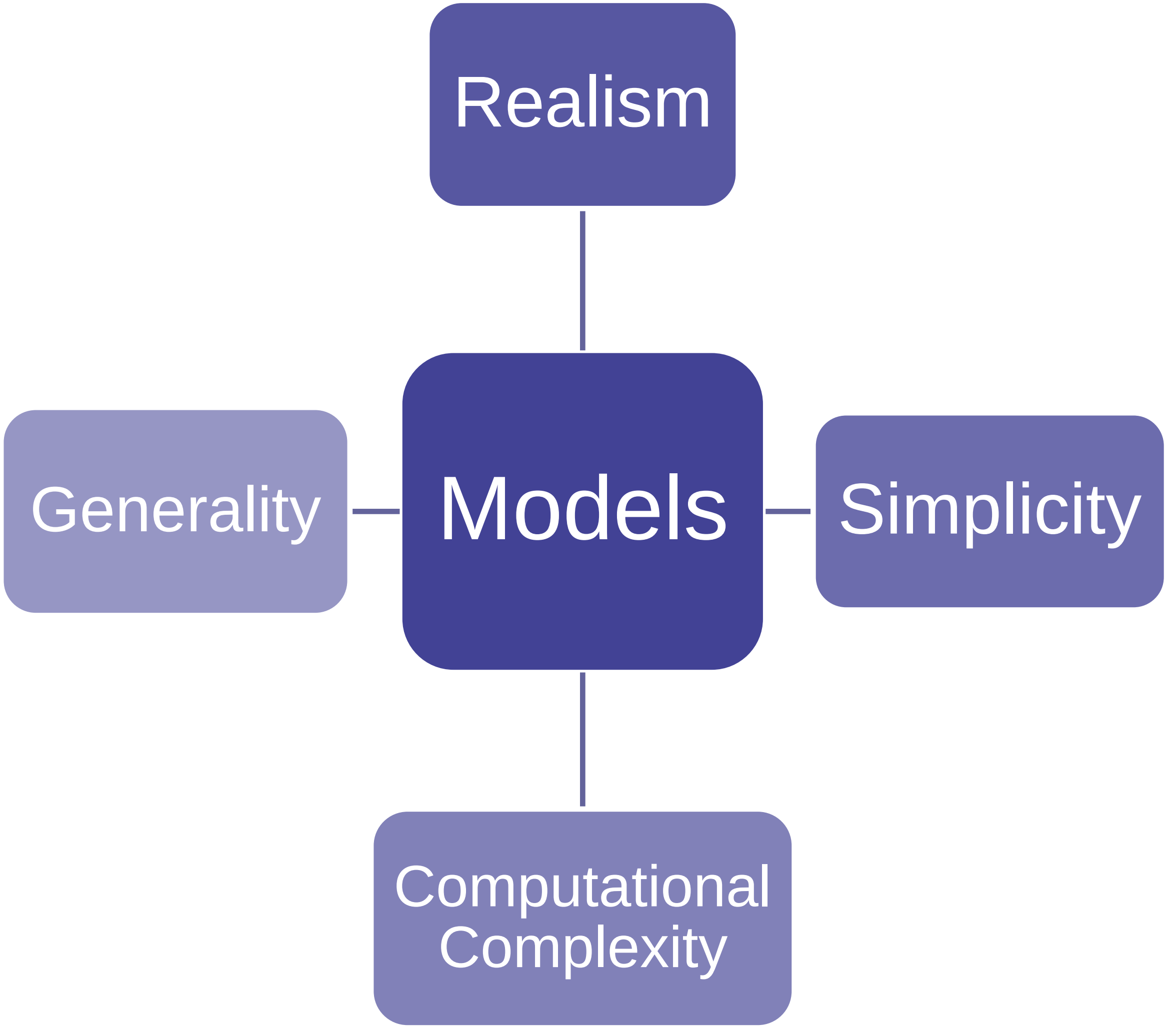


$$w(u,v) = a_v(u) + a_u(v)$$

Weighted degree of u

MODELING SINR WITH GRAPHS

Which problem → Which model?



Rethinking graphs for representing interference

- Graphs are preferable to working directly with SINR
  - Less conceptual complexity
  - Simplifies description
  - Lots of theory already established

- How well can graphs work?

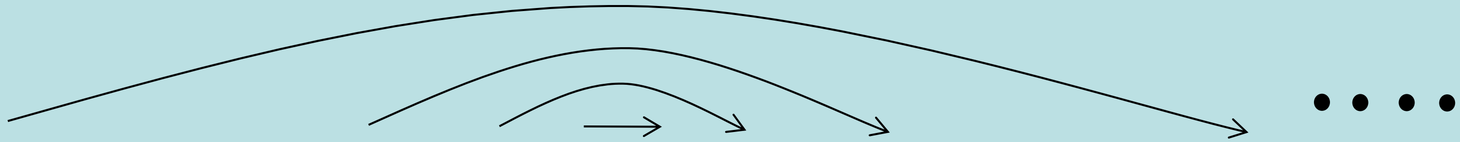
- What does it mean to „represent SINR relationship“?

First success

- If the links are of similar lengths (Delta = constant), then Unit Disc Graphs are a good approximation [H, ESA‘09]

- Additional requirement:
  – Maintain bounded contention in every „neighborhood“
  – Decay algorithm would fail in this respect

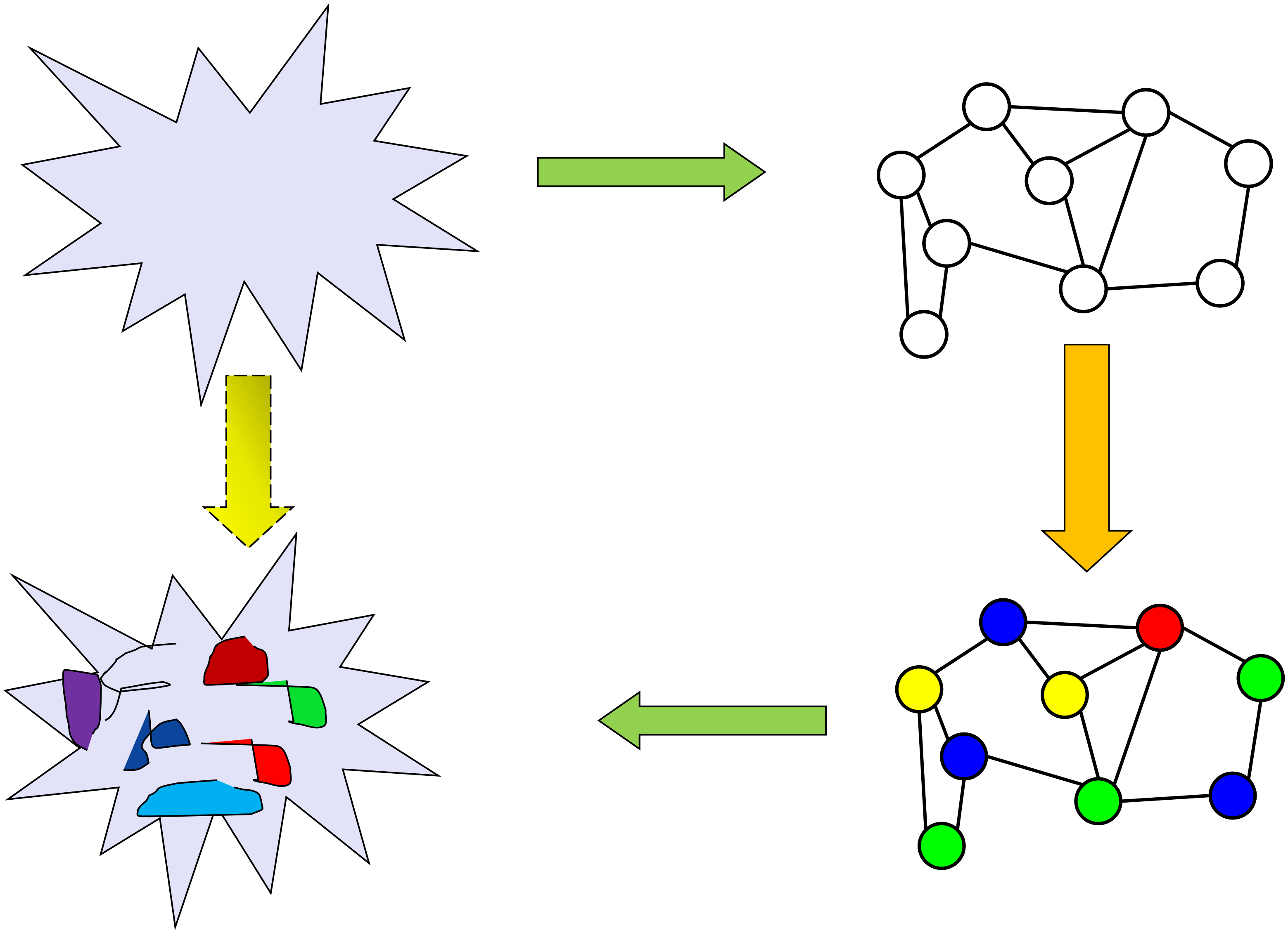
Disc Graphs Fail



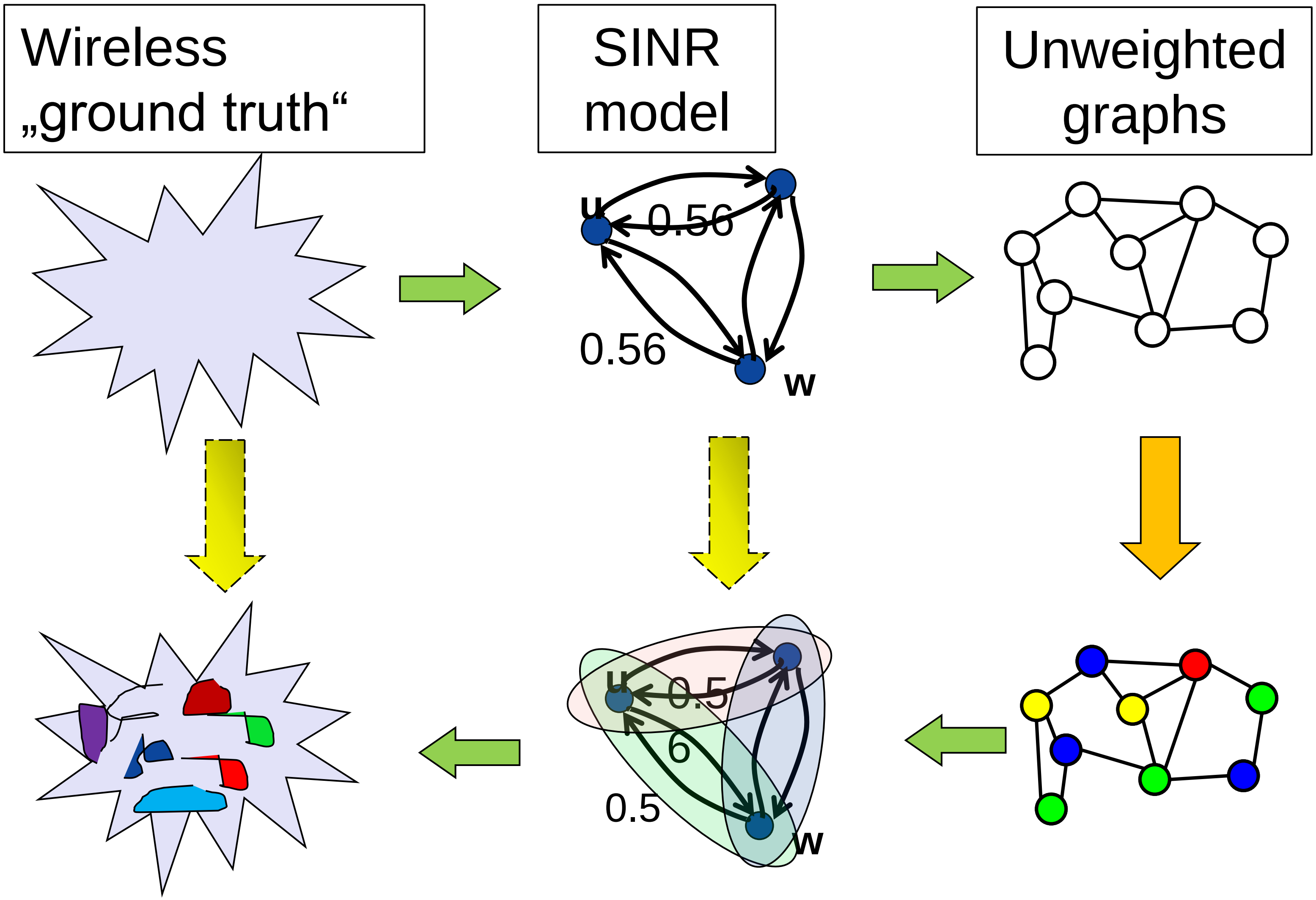
Length of link i = 2^i [Moscibroda, Wattenhofer 2006]

Feasible set, but forms a clique in any disc graph

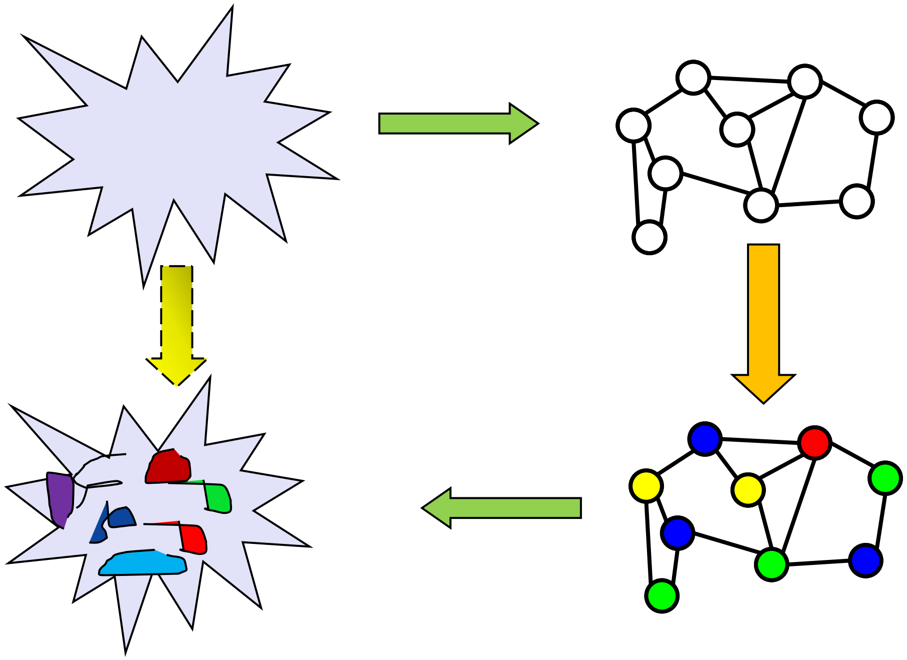
Approach: Abstract, solve, map back



Hierarchies of abstraction



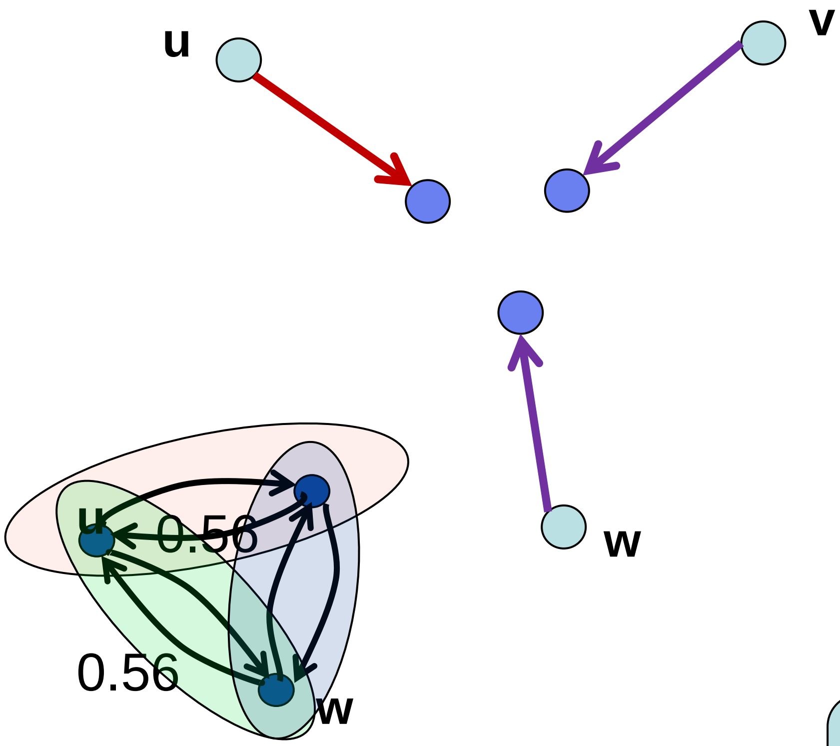
Price of abstraction



- Price of abstraction :

How much you lose by solving the abstracted problem (rather than solving directly)

Representing link scheduling with a graph



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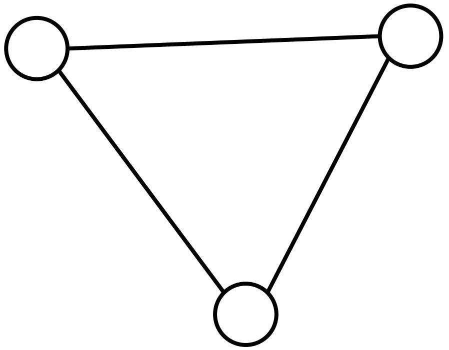
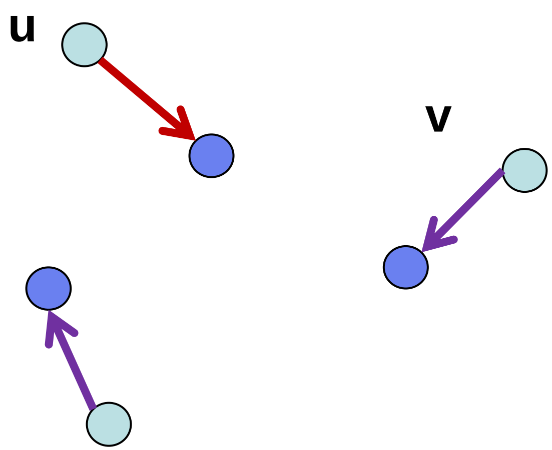
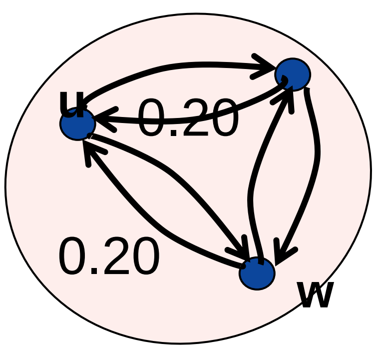
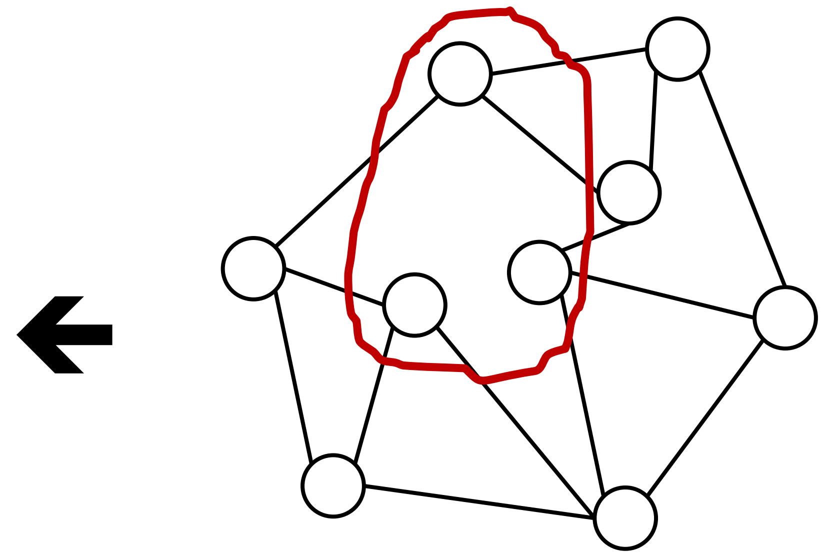


When should there be an edge?

Requirement I: Feasibility



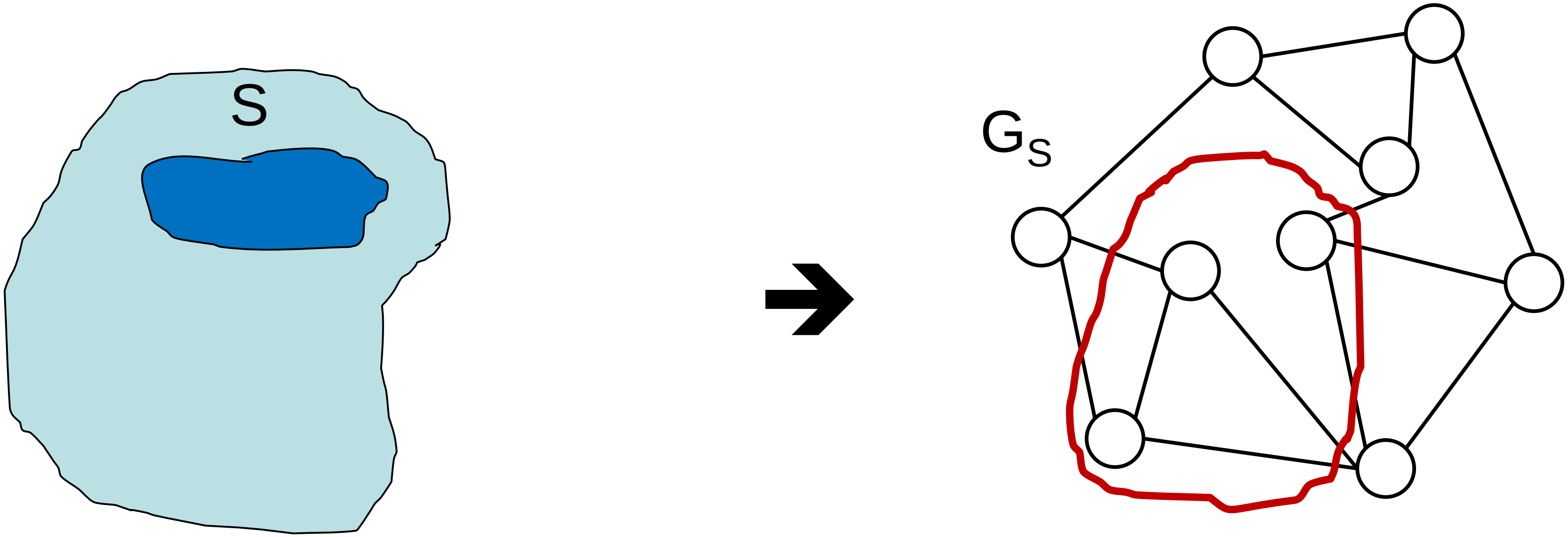




Independent sets should be <u>feasible</u>

valid coloring of G
$\Rightarrow$ valid scheduling

Requirement II: Near-independence



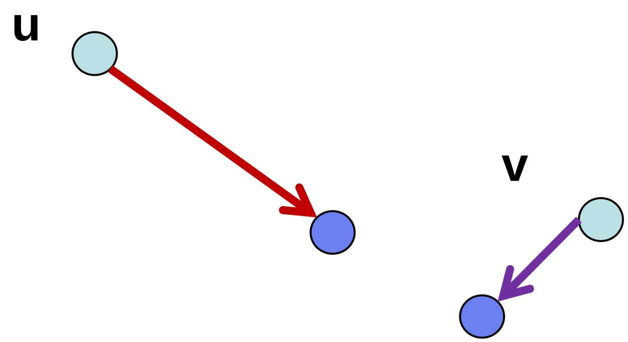
Feasible linksets should be „nearly independent“ in G

S feasible $\Rightarrow \chi(G_S)$ small

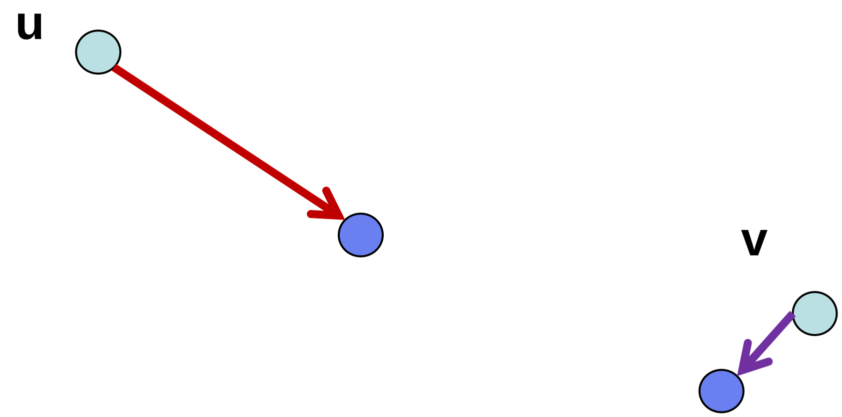
Small cost of abstraction!

Possible graphs schemas (that fail)

- Pairwise conflicts
  - $d(u, v) \leq c \cdot \min(|u|, |v|)$
  - Too relaxed (fail feasibility)
  - One of the links will always be infeasible



- Disc graphs
  - $d(u, v) \leq c \cdot \max(|u|, |v|)$
  - Too conservative (high cost)



- Solution: Interpolate?

Conflict graph representations [H,Tonoyan, STOC'15]

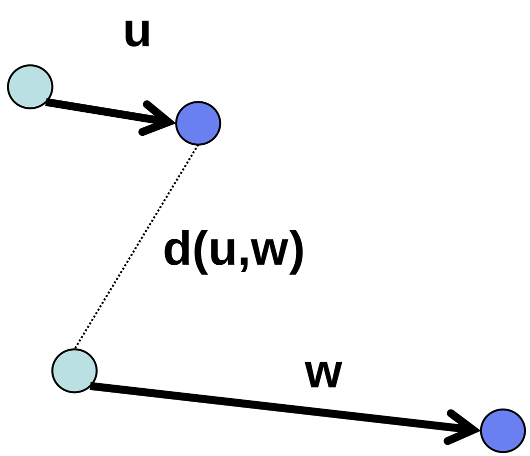
Adjacency predicate:

$$d(u,w) \le f\left(\frac{|w|}{|u|}\right)|u|,$$

(f monotone)

f linear : disc graphs
f const : pairwise SINR

(w is longer than u)



All such graphs have constant *inductive independence,* which allows for constant-factor approximation of our problems

Conflict graph representations [H,Tonoyan, STOC'15]

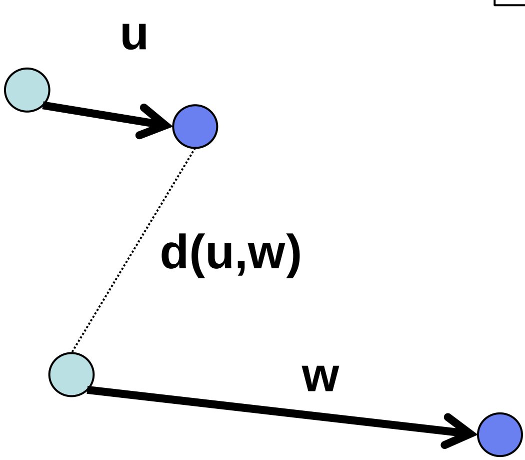
Adjacency predicate:

$$d(u,w) \le f\left(\frac{|w|}{|u|}\right)|u|,$$

(f monotone)

f linear : disc graphs
f const : pairwise SINR

Feasibility holds for $f(x) = \Omega(\log x)$



Cost of abstraction is $f^*(x)$,
the iterated application of f

For $f = \log$, the cost is $O(\log^* \Delta)$
Δ = Diversity in link lengths
($\log^* \Delta$ is always less than 4...)

Implications

- $O(\log^* \Delta)$-approximation of *Link Scheduling*

Implications : Other problems

- Sandwiching property: Given set of links, we form *two* graphs G_1 and G_2 s.t. for all $S \subseteq L$,

$$\chi(G_1(S)) \leq Sched(S) \leq \chi(G_2(S))$$

- Nearly all other scheduling problems can be solved:
  - Multi-channel multi-antennas
  - Multi-hop scheduling with fixed paths
  - Maximum multiflow
  - Maximum concurrent multiflow...
- Other applications:
  - Online algorithms (admission control)
  - Spectrum auctions

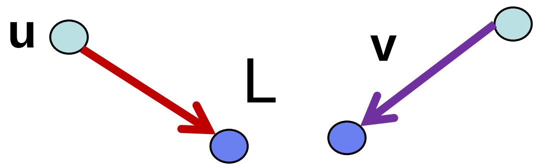
How far can we go? Limits of solvability

- No (theoretical) study is complete without exploring the limits of the doable.

- Can we show that no conflict graph schema can perform better?

Axioms for conflict graph representations

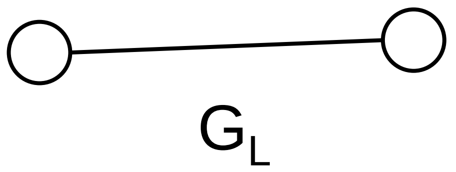
- Defined by pairwise relationship of links
- Independent of position and scale (scale-free)
- Monotonic with increasing distances
- Symmetric w.r.t. sender and receiver



Every conflict graph schema is sandwich by formulations

$$d(u,w) \leq f\left(\frac{|w|}{|u|}\right)|u|,$$

where f is a monotone function



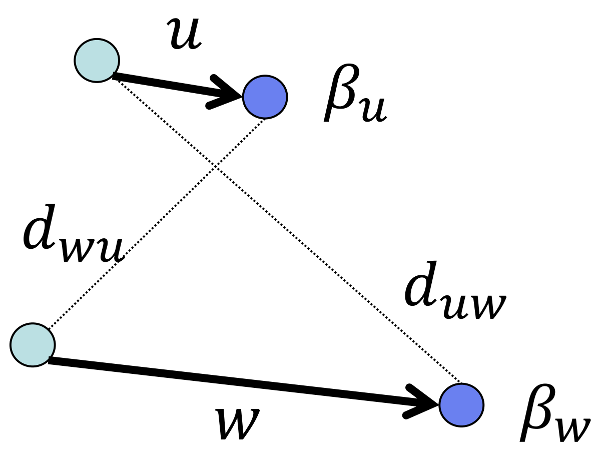
Limitation results

- A. Any conflict graph representation incurs a $\Omega(\log^*(\Delta))$ factor → Price of abstraction is $\Theta(\log^*(\Delta))$
  - i) For every monotone f, there is an instance that is feasible but whose conflict graph is a clique and requires $\Omega(f^*(\Delta))$ colors
  - Ii) For $f = O(\log^{1/\alpha} n)$, there is an instance whose conflict graph is independent, but requires $\Theta(\log^*(\Delta))$ slots to schedule.
- Builds on a construction of [H, Mitra, SODA'12]

- B. No approximation in terms of *n* is possible.

- C. Requires Euclidean or doubling metrics

Conflict graph w/ data rates [H,Tonoyan ICALP'17]



Effective length of link u : $\ell_u = |u| \cdot \beta_u^{1/\alpha}$

Adjacency predicate:

$$d_{uw} d_{wu} \leq f\left(\frac{\ell_u}{\ell_w}\right) \ell_u \ell_w,$$

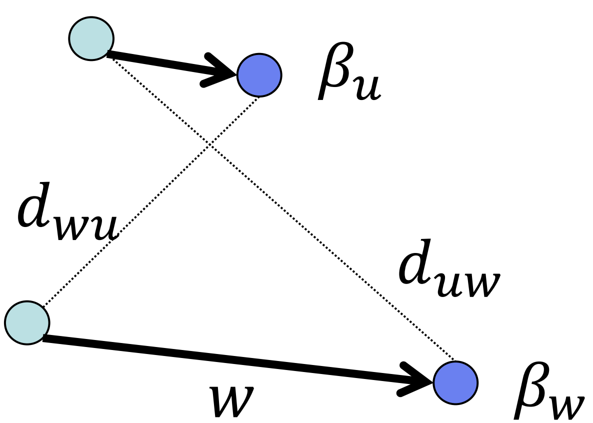
(for $\ell_u \geq \ell_w$)

Conflict graph w/ data rates [H,Tonoyan ICALP'17]

Adjacency predicate:

$$d_{uw} d_{wu} \leq f\left(\frac{|w|}{|u|}\right) |u||w|,$$

(f monotone)



For $f(x) = \sqrt[c]{x}$, the cost of abstraction is $O(\log\log\Delta)$

Achievable with *oblivious* power: depends only on link length.

$\Theta(\log\log\Delta)$ is also best possible

Handling Bit-Rates and Utilities

„It‘s the rate.... stupid“

- Attain $\Theta(\log\log\Delta)$-approximation for capacity problems involving bitrates

- Can handle arbitrary fixed or variable bitrates
  - Graphs have the same desirable properties (inductive indep.)

Further applications [H,Tonoyan, unpub.]

- The original „*complexity of connectivity*“ problem of [Moscibroda, Wattenhofer, 2006]:
  - Connect all nodes into a (convergecast) tree
  - Choose power assignment
  - Schedule the edges, in the fewest number of slot.
- Corresponds to achievable *rate of aggregation*

- $O(\log^* \Delta)$ time slots suffice
  - Uses the Minimum Spanning Tree
- $\Omega(\log^* \Delta)$ slots necessary for scheduling an MST

NOW WHAT?

Challenge: Robustness, dynamicity

- Robustness, dynamicity

- Heterogeneity

Challenge: New technologies

- SINR-like models correct only for uncorrelated signals
- Using alignment of signals, can achieve:
  - Directional transmissions
  - = Beamforming
  - More diversity => avoid weak links
  - Multiple Receive/transmit => Higher bandwidth
  - Transmit multiple message streams (MIMO)
- High frequency channels

Challenge: Modeling

Open questions

- Still have not answered the question if truly constant-factor approximation is possible
- Can we leverage this graph representation further?
- Distributed algorithms
- Handling dynamic situations
- New modes of communication (*interference alignment*)
  - Beamforming, MIMO, *cooperative, cancellation*,...

Open questions

- Uniform power
  - Only one power level
  - Should be „easier“, but we understand it less analytically
- Understanding SINR
- The role of CS theory in wireless computing

Take-home message

- Graphs are maybe (more than) fine !
  - Question of the level of abstraction

- The meta question of the right model

Collaborators

- Tigran Tonoyan

- Eyjólfur Ásgeirsson

- Roger Wattenhofer (ETH)

- Stephan Holzer (MIT)

Experimental group at RU:

- Helga Gudmundsdottir
- Ýmir Vigfusson
- Joe Foley

Alumni:

- Pradipta Mitra (Google)

- Marijke Bodlaender

Thank you

Other contributions

- Q: What about results that hold only in the plane, like most distributed algorithms?

- A: These generally carry over also to decay spaces that form *doubling metrics*
  - We introduce a new term that represents the cumulative interference from a uniformly spread set of nodes
  - If this is constant bounded, then most algorithms work

- Q: Which algorithms don‘t work?
- A: Those that depend on Euclidean properties:
  - SINR diagrams
  - Algorithms using angles

Open Issues

- Temporal variability, dynamicity
  - Major issue, largely untouched
  - How is it dependent across time? What time window is static?
- Interference alignment
  - Alignment, cancellation, beamforming

Take-home message

- When chosen with care, graphs are surprisingly good.
- Constant approximations for Scheduling probably not possible.

- Results in the geometric SINR model carry over to general, realistic settings....

- ... as long as brittle assumptions are avoided

Modeling Wireless Communications Algorithmically

- When is a wireless transmission successful?
- How much communication can take place simultaneously?
- How to *schedule* it, to maximize thruput, minimize latency

Core Questions

- How to *schedule* it, to maximize thruput, minimize latency

- How do we study/analyze such algorithms?

- How do we model signal reception, propagation?

Extras

- Brilliance
  - https://www.youtube.com/watch?v=-ciFTP_KRy4