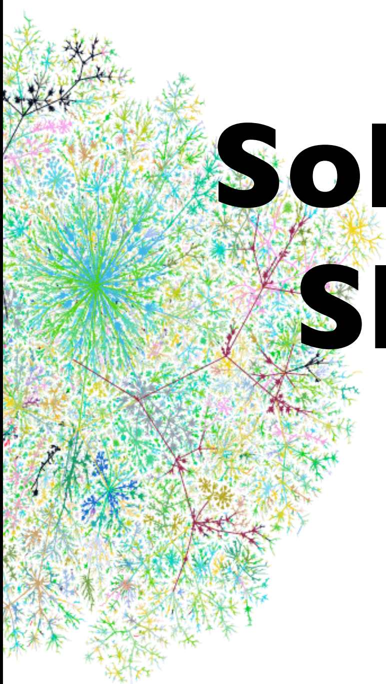
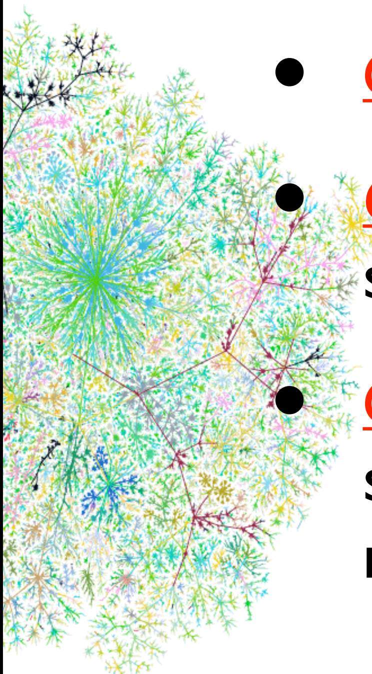


Solving Graph Problems via Sketching and Streaming

Andrew McGregor

University of Massachusetts

Streaming



- *Input* Observe stream of edge insertions/deletions.
- *Goal* Using small memory, compute properties of the graph.
- *Classic Stream Results* Estimate statistics of numerical streams, such as quantiles, frequency moments, heavy hitters...
- *Graph Streams* Growing body of work on problems with more structure: distances, cuts, eigenvalues, random walks, clustering, matchings, dense components, vertex covers, hitting sets…

Survey: SIGMOD Record 2014
http://people.cs.umass.edu/~mcgregor/papers/13-graphsurvey.pdf

Class Notes: CMPSCI 711, UMass
https://people.cs.umass.edu/~mcgregor/courses/CS711S18/

Graph Stream Algorithms: A Survey

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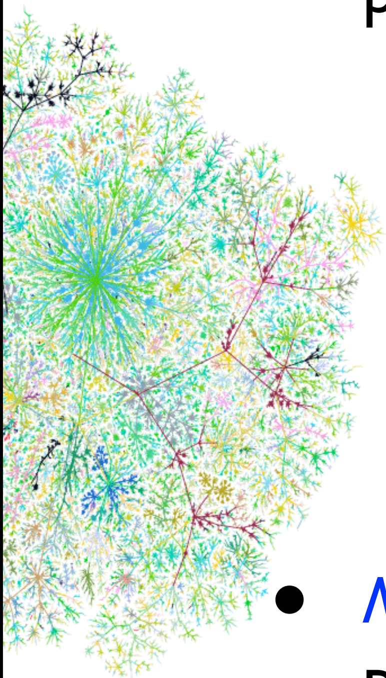
Sketching



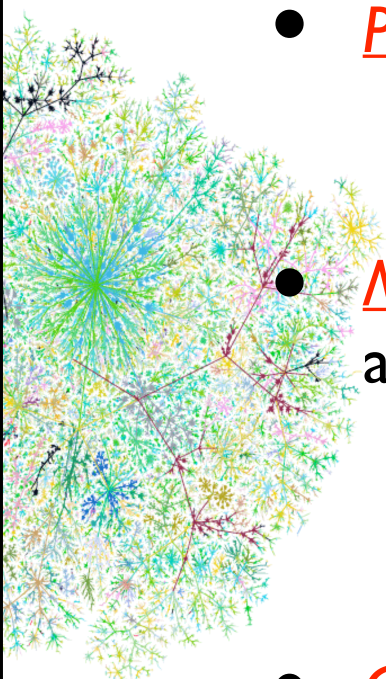
- *Random linear projection* M: $\mathbb{R}^n \rightarrow \mathbb{R}^k$ where $k \ll n$ that preserves properties of any $v \in \mathbb{R}^n$ with high probability.

$$\begin{bmatrix} & & M & & \end{bmatrix} \begin{bmatrix} \\ v \\ \\ \end{bmatrix} = \begin{bmatrix} Mv \end{bmatrix} \longrightarrow \text{answer}$$

- *Many results* for numerical statistics and basic geometric properties... *extensive theory* with connections to hashing, compressed sensing, dimensionality reduction, metric embeddings... *widely applicable* since embarrassingly parallelizable and suitable for stream processing.

? *Question* What about analyzing massive graphs via sketches?

Summary



- *Preliminaries* L_0 sampling and densest subgraph.

  "You can always do uniform sampling; sometimes it suffices."

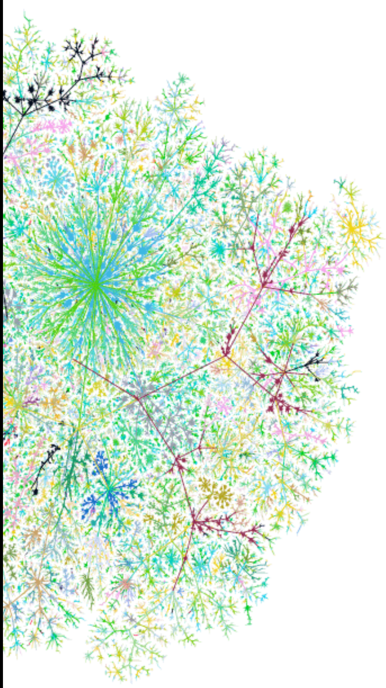
- *Matching Story* Using sketches to compute exact matchings, approximate matchings, and planar matchings.

  "Sketches enable interesting types of non-uniform sampling that are useful for graph problems."

- *Connectivity Story* Using sketches to analyze edge and node connectivity, build cut and spectral sparsifiers etc.

  "Homomorphic compression: sketch first, compute later."

- *Other Stories* Four small-space results we didn't have space for.



part 0: **<u>Preliminaries</u>**

part 1: **Matchings**

part 2: **Connectivity**

part 3: **Other Stories**

Uniform Edge Sampling via Sketches

- *L_0 sampling* Can use sketches to uniformly sample an edge from the graph stream using O(polylog(n)) space.

  Jowhari, Saglam, Tardos [PODS 11], *Kapralov et al.* [FOCS 17]

- Easy if there's only edge insertions but non-trivial with insertions and deletions. Can treat result as a blackbox but will be important that the result is via linear sketches.

Application to Densest Subgraph

- Given a graph G, the *density* of a set of nodes S is:

$$D_S = \frac{\# \text{ of edges with both endpoints in } S}{\# \text{ of nodes in } S}$$

- <u>*Previous Result*</u> $2+\varepsilon$ approx of max density D^* in $\tilde{O}(\varepsilon^{-2} n)$ space.

  Bhattycharya et al. [STOC 15], *Bahmani et al.* [PVLDB 12]

- <u>*Our Result*</u> One pass $1+\varepsilon$ approximation using $\tilde{O}(\varepsilon^{-2} n)$ space:

  Use L_0 sampling to uniformly sample $\tilde{O}(\varepsilon^{-2} n)$ edges. Let $\check{D}_S$ be estimate of D_S based on sampled edges. Return $\max_S \check{D}_S$.

  McGregor, Tench, Vorotnikova, Vu [MFCS 15]

- <u>*Analysis*</u> For any set of k nodes S, with probability $1-n^{-2k}$,

$$\check{D}_S = D_S \pm \varepsilon D^*$$

  Use union bound over $O(n^k)$ subsets of size k for each k.

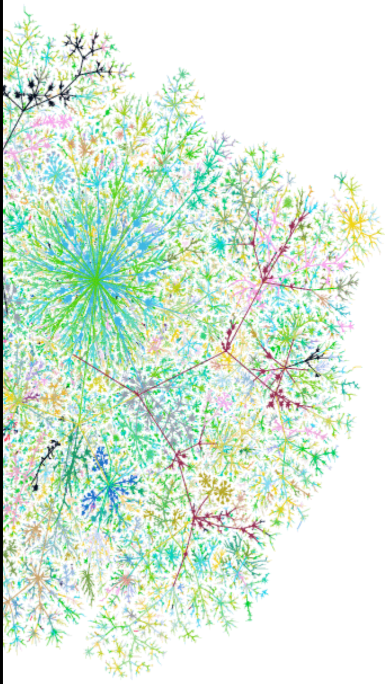
  see also *Mitzenmacher et al.* [KDD 15], *Esfandiari et al.* [SPAA 16]

More details about L_0 sampling

- L_0 Sampling: There's a random $M: \mathbb{R}^N \to \mathbb{R}^{\text{polylog } N}$ such that for any $a \in \mathbb{R}^N$, we can find random non-zero entry of a from Ma whp.
- Entry in i^{th} row of M is 1 w/p 2^{-i+1}. Some entry of Ma probably corresponds to single entry of a

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ 0 \\ 0 \\ y \\ z \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} x+y+z \\ x+z \\ y \\ 0 \end{pmatrix} \begin{array}{l} \leftarrow \text{Too many} \\ \leftarrow \text{Too many} \\ \leftarrow \text{Just right} \\ \leftarrow \text{Too few} \end{array}$$

Detail: Need some extra tricks to a) recognize when entry of Ma corresponds to a single entry of a and b) determine the index of this entry



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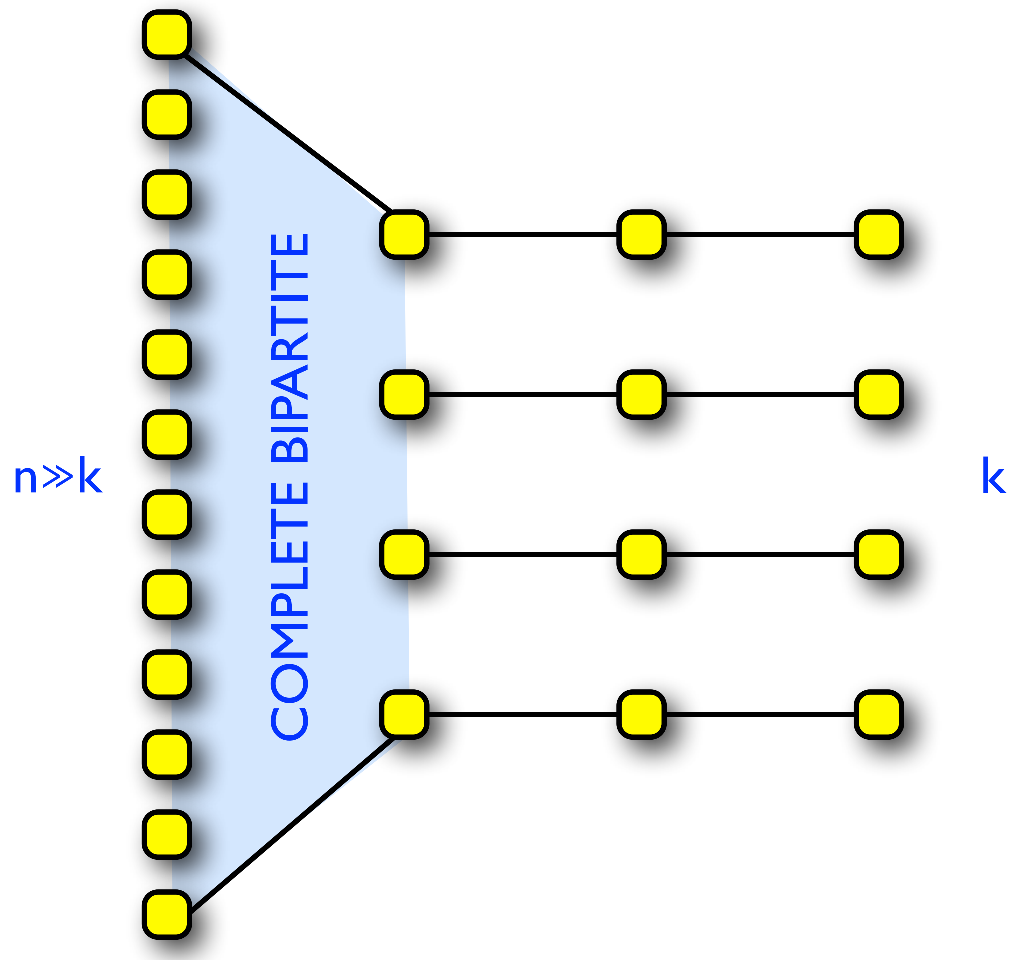
Sometimes Uniform Sampling Isn't Enough...



Need to uniformly sample $\Omega(kn)$ edges before we find a matching of size 2k.

What other types of sampling a) are useful for solving graph problems and b) can be supported on dynamic graph streams via sketches?

Matchings Story

- *Exact small matchings:* If matching has size ≤ k can find it exactly in $\tilde{O}(k^2)$ samples. Gives optimal stream algorithm.

  Chitnis et al. [SODA 16], *Bury et al.* [Algorithmica 18]

- *Approximate matching:* Find t-approx matching in $\tilde{O}(n^2/t^3)$ samples. Gives optimal stream algorithm.

  Chitnis et al. [SODA 16], *Assadi, Khanna, Li, Yaroslavtsev* [SODA 16]
  related: *Konrad* [ESA 15], *Bury, Schwiegelshohn* [ESA 15]

- *Planar matching:* Can 5+ε approx matching size in planar graphs using $\tilde{O}(n^{4/5})$ space. Polylog space suffices if there are no edge deletions.

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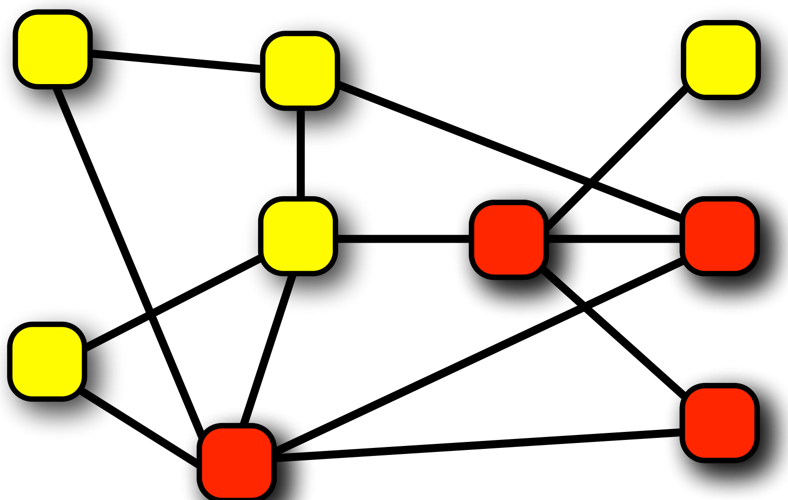
Sample-Nodes-And-Pick-Edge Sampling

- *To get a single SNAPE Sample:*
  - Sample each node with probability 1/k and delete rest
  - Pick a random edge amongst those that remain.

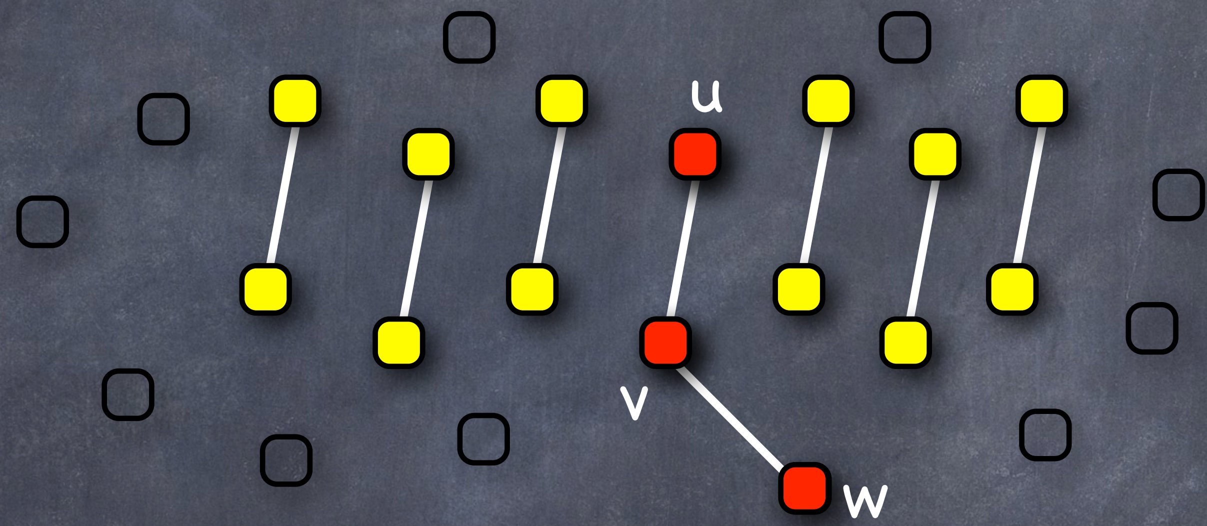


- *Theorem* If G has max matching size k, then $O(k^2 \log k)$ SNAPE samples will include a max matching from G.

Chitnis et al. [SODA 16], related: *Bury, Schwiegelshohn* [ESA 15]

Why SNAPE Sampling Works...

- Consider a maximum matching M of size k and focus on arbitrary edge {u,v} in this matching.



- u and v only endpoints of M sampled with prob. $\Omega(k^{-2})$.
- Hence, when we pick one of the remaining edges it's either {u,v} or another edge that's equally useful.
- Take $O(k^2 \log k)$ samples; apply analysis to all edges.

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Estimating Matching Size in Planar Graphs

- A graph has arboricity α if any induced subgraph on r nodes has at most αr edges. For a planar graph $\alpha=3$.
- *Lemma:* match(G)/$(2+\alpha) \leq A \leq$ match(G) where A is total edge weight if each edge uv gets weight

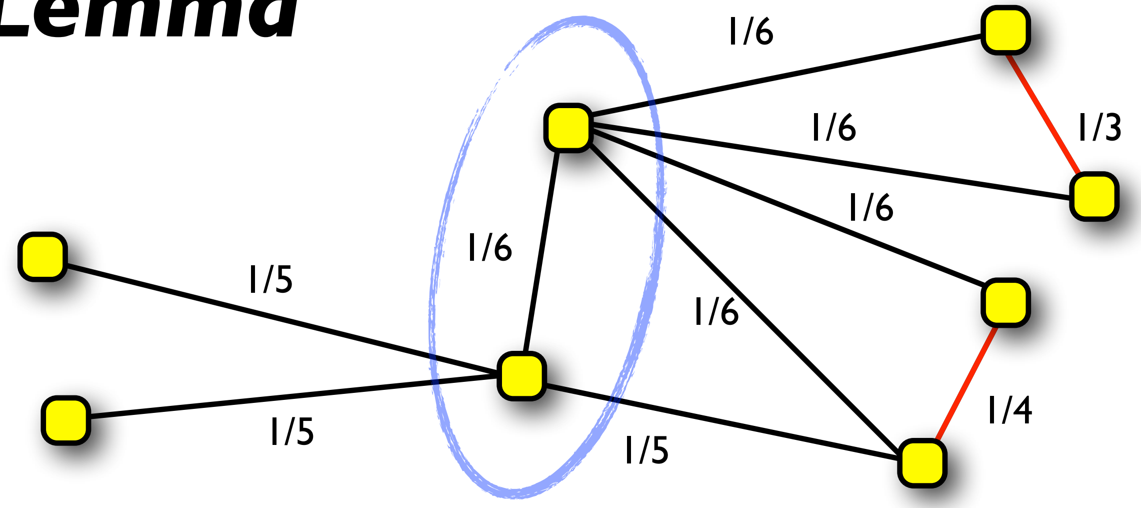
$$x_{uv} = \min\left(\frac{1}{\deg(u)+1}, \frac{1}{\deg(v)+1}\right)$$

- *Thm:* Can $2+\alpha+\varepsilon$ approximate match(G) using $\tilde{O}(n^{4/5})$ space:

  If match(G)$\leq n^{2/5}$, can find exactly using earlier algorithm.

  Otherwise, evaluate A on random set of $\approx n^{4/5}$ nodes.
- *Corollary:* $5+\varepsilon$ approx for planar graphs.

Proof of Lemma



- The edge weights are a fractional matching, i.e., for any node u:

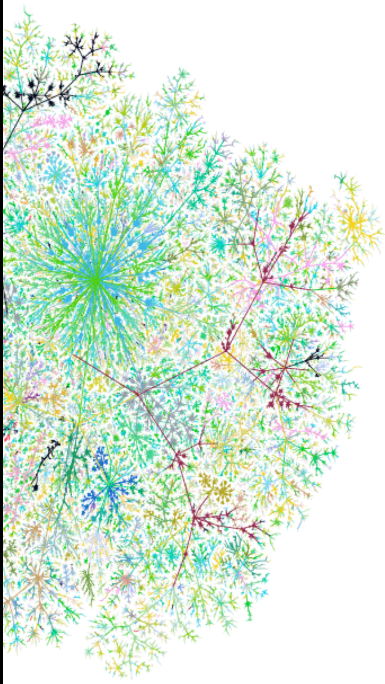
$$\sum_{v \in \Gamma(u)} x_{uv} \leq \sum_{v \in \Gamma(u)} \frac{1}{\deg(u)+1} < 1$$

- To prove total weight ≤ match(G): Use Edmond's matching polytope thm since weight on subgraph of r nodes is ≤(r-1)/2.

- To prove total weight ≥ match(G)/(2+α):

  Total weight of edges incident to "high degree" vertices H at least |H|/(2+α) and all other weights are at least 1/(2+α).

  Matching size is at most |H| + "edges not incident to H"



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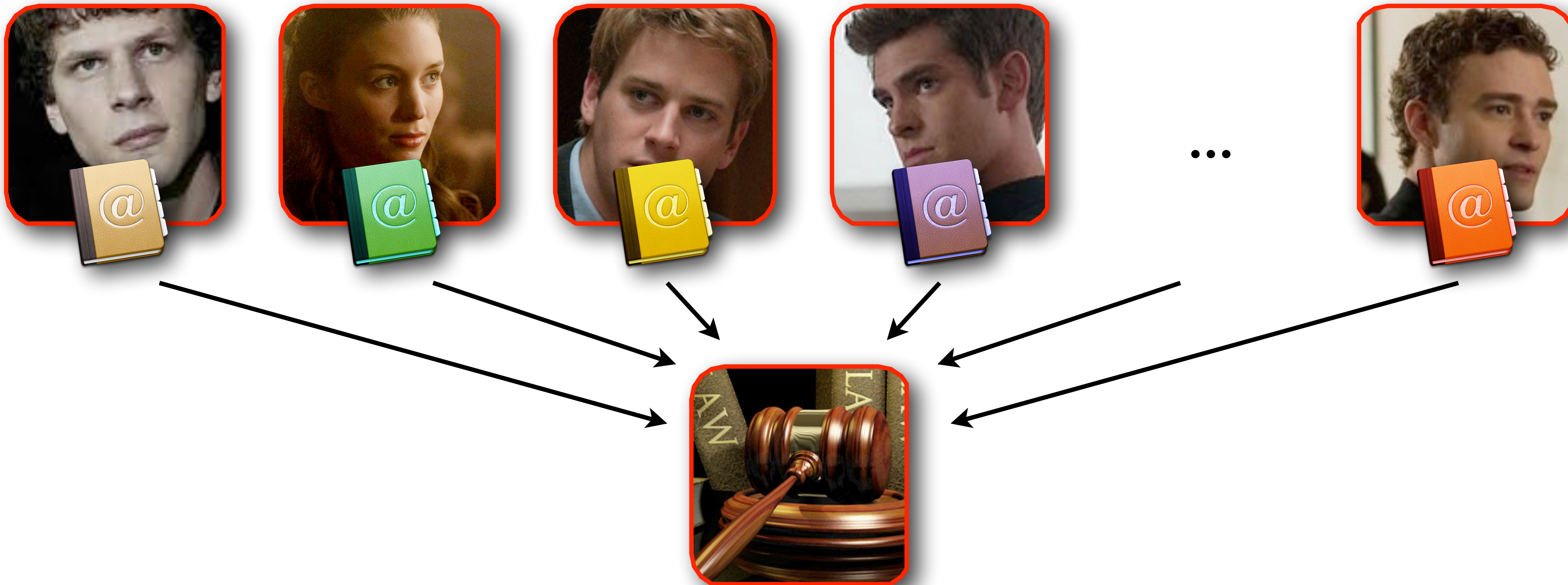
Computing with sketches...



- <u>*Problem:*</u> n people each with a list of their friends amongst the group. In parallel, each sends a small number of bits to a central player who must determine if underlying graph is connected.
- <u>*Thm:*</u> $O(\log^3 n)$ bits from each player suffices.
- Any approach just using sampling fails… e.g., players can't distinguish bridge edges from other edges in the graph.

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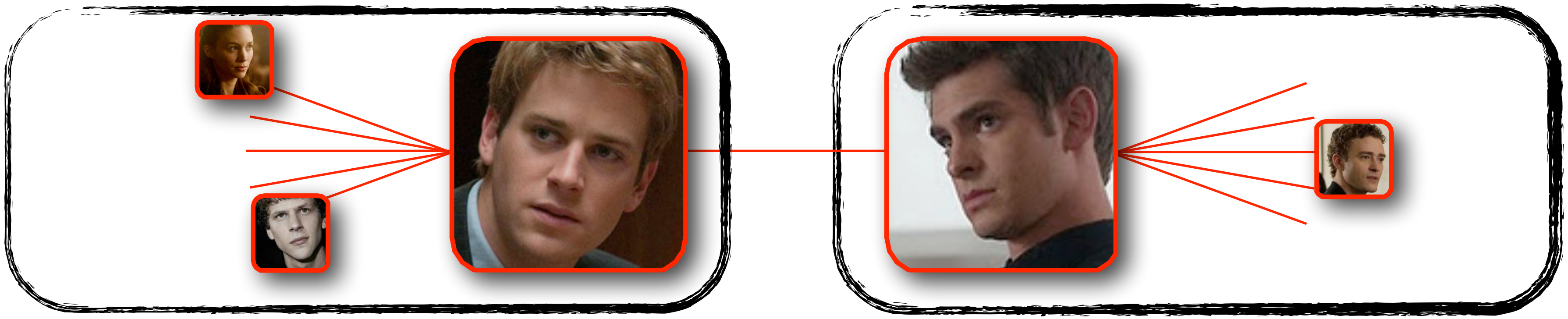

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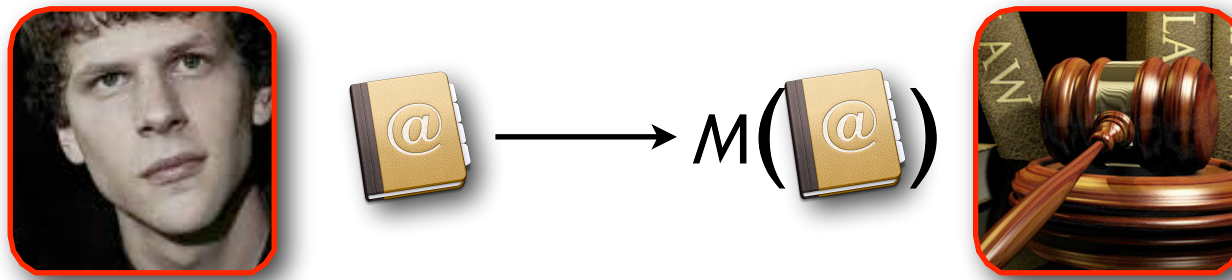
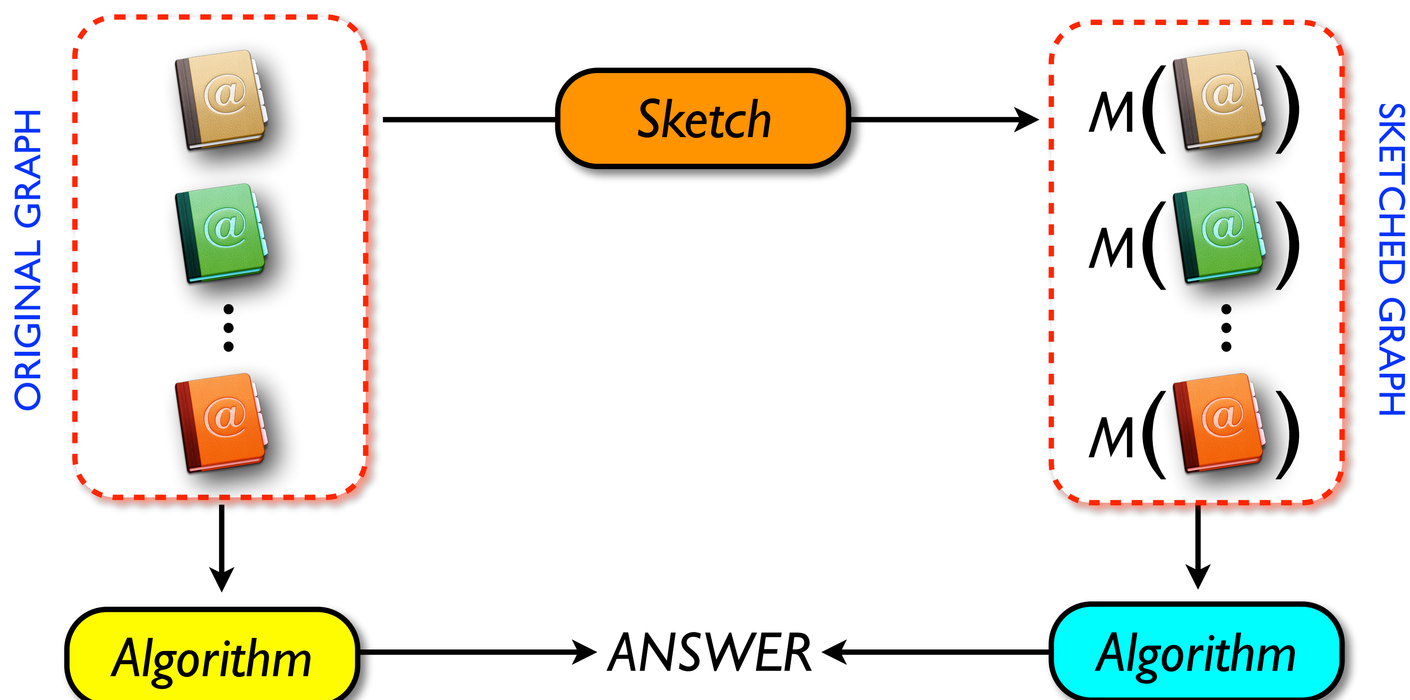
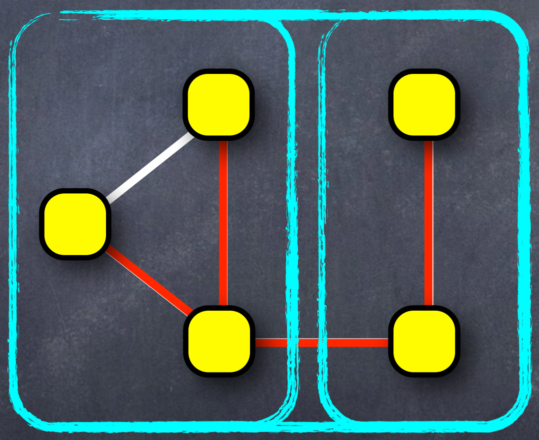


- Players send carefully-designed sketches of address books.
- *Homomorphic Compression:* Instead of running algorithm on original data, run algorithm on sketched data.



Ingredient 1: Basic Algorithm

- Algorithm (Spanning Forest):
  - For each node: pick incident edge
  - For each connected component: pick incident edge
  - Repeat until no edges between connected comp.

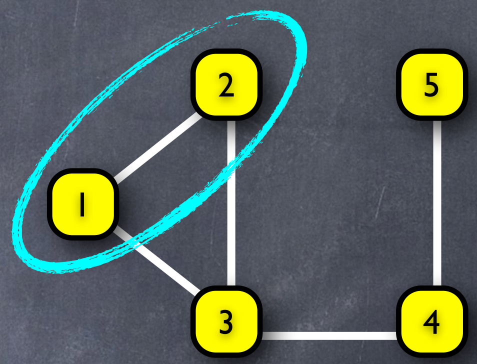


- Lemma After O(log n) rounds selected edges include spanning forest.

Ingredient 2: Sketching Neighborhoods

- For node i, let a_i be vector indexed by node pairs. Non-zero entries: $a_i[i,j]=1$ if $j>i$ and $a_i[i,j]=-1$ if $j<i$.

$$
\begin{array}{r@{\,}c}
 & \begin{array}{cccccccccc} \{1,2\} & \{1,3\} & \{1,4\} & \{1,5\} & \{2,3\} & \{2,4\} & \{2,5\} & \{3,4\} & \{3,5\} & \{4,5\} \end{array} \\
\mathbf{a}_1 = & \left(\begin{array}{cccccccccc} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \\
\mathbf{a}_2 = & \left(\begin{array}{cccccccccc} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \\
\mathbf{a}_1 + \mathbf{a}_2 = & \left(\begin{array}{cccccccccc} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{array} \right)
\end{array}
$$



- Lemma For any subset of nodes $S \subset V$, non-zero entries of $\sum_{j \in S} a_j$ are edges across cut $(S, V \setminus S)$
- Player j sends $M(a_j)$ where M is "L_0 sampling" sketch.

Recipe: Sketch & Compute on Sketches

- Player with Address Books: Player j sends Ma_j
- Central Player: "Runs Algorithm in Sketch Space"
  - Use Ma_j to get incident edge on each node j
  - For i=2 to log n:
    - To get incident edge on component $S \subset V$ use:

$$\sum_{j \in S} M\mathbf{a}_j = M(\sum_{j \in S} \mathbf{a}_j) \longrightarrow \text{non-zero elt. of } \sum_{j \in S} a_j = \text{edge across cut}$$

Detail: Actually each player sends log n independent sketches M_1a_j, M_2a_j, ... and central player uses M_ia_j when emulating i^{th} iteration of the algorithm.

Connectivity Story

- *Connectivity:* Test k-edge connectivity with $\tilde{O}(k)$ bit sketches.

  Ahn, Guha, McGregor [SODA 12]

- *Cut sparsification:* Estimating size of every cut up to $(1+\varepsilon)$ factor with $\tilde{O}(\varepsilon^{-2})$ bit sketches.

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- *Spectral sparsification:* Estimating eigenvalues up to $(1+\varepsilon)$ factor with $\tilde{O}(\varepsilon^{-2})$ bit sketches.

  Kapralov, Lee, Musco, Musco, Sidford [FOCS 14]

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Extending to k-Edge-Connectivity

Basic Algorithm

- Algorithm: For i=1 to k:

  Let F_i be spanning forest of $G(V,E-F_1-\ldots-F_{i-1})$

- Lemma: $F_1+\ldots+F_k$ is k-edge-connected iff G is.

Emulation in Sketch Space

- Sketch: Simultaneously construct k independent connectivity sketches $M_1(G)$, $M_2(G)$, …, $M_k(G)$.
- Run Algorithm in Sketch Space:
  - Use $M_1(G)$ to find a spanning forest F_1 of G
  - Use $M_2(G)-M_2(F_1)=M_2(G-F_1)$ to find F_2
  - Use $M_3(G)-M_3(F_1)-M_3(F_2)=M_3(G-F_1-F_2)$ to find F_3…
- Extension: Can recover a set of "weak" edges whose removal leaves connected components with min-cut > k.

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Cut Sparsification

Basic Algorithm

- Algorithm:

  Let G_0, G_1, … $G_{O(\log n)}$ where $G_0=G$, and G formed from G_{i-1} by deleting each edge with probability 1/2

  Let W_i be set of edges in G_i whose removal leaves a graph where non-zero cuts have $\Omega(\varepsilon^{-2} \log n)$ edges.

- Lemma: Whp can estimate all cuts in G_i given W_i and G_{i-1}… can estimate all cuts in G given W_0, W_1, … $W_{O(\log n)}$

Emulation in Sketch Space

- Sketch: For each i, use the k-edge-connectivity sketches of G_i to find W_i.

Connectivity Story

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k-Node-Connectivity

Basic Algorithm

- Algorithm:

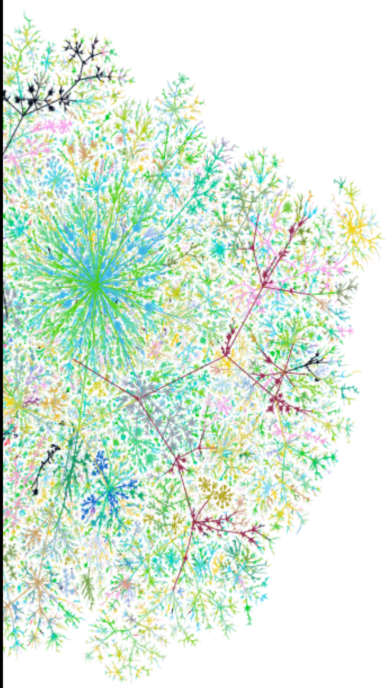
  Let $H_1, H_2, \ldots H_r$ where $r=k^3 \log n$ and H_i is induced subgraph on random set of n/k nodes.

  Let F_i be a spanning forest of H_i.

- Lemma: Whp, $F_1+F_2+\ldots+F_r$ is k-node connected iff G is.

Emulation in Sketch Space

- Sketch: Construct connectivity sketch for each H_i, and use this to find F_i.



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Other Stories

- *Other Problems* Max-cut, clustering coefficients and triangles, degree distribution, correlation clustering, independent sets
  Cormode et al. [ICML 15], *Bulteau et al.* [Algorithmic 16], *McGregor et al.* [PODS 16], *Cormode, Dark, Konrad* [arXiv 17], *Kapralov et al.* [SODA 17]

- If stream consists of m subsets of nodes…

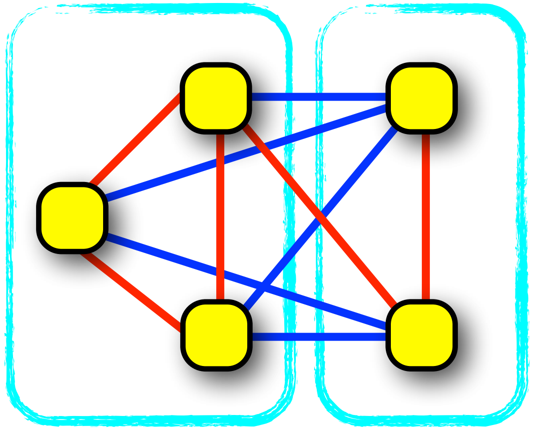
  - *Set-cover* Θ(mn/t) space for t-approx in one pass also tight space/pass/approx. trade-offs. *Assadi et al.* [STOC 16] *Chakrabarti, Wirth* [SODA 16], *Har-Peled et al.* [PODS 16]

  - *Hitting set* Optimal hitting of size k in $\tilde{O}(k^d)$ if all sets have size at most d. *Chitnis et al.* [SODA 16]

  - *Max k-Coverage* 1-ε approx in $\tilde{O}(m\varepsilon^{-2}\min(\varepsilon^{-1},k))$ space or 0.5 approx in $\tilde{O}(k)$ space.
    McGregor, Vu [ICDT 17], *Bateni et al.* [SPAA 17], *Assadi* [PODS 17]

Clustering + Maximal Independent Set

- Consider a complete graph where edges are labelled attractive or repulsive. Given a node partition, an attractive edge is sad if it is cut and a repulsive edge is sad if it is not cut.



- *Correlation Clustering* Find partition minimizing # sad edges.

  See tutorial *Bonchi, Garcia-Soriano, Liberty* [KDD 14]

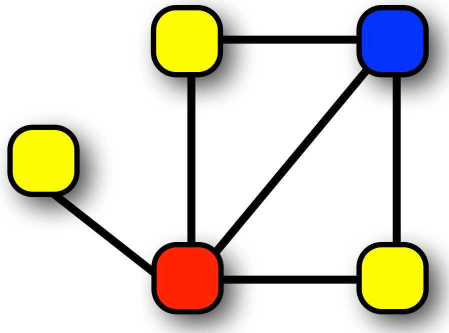
- *3-Approx Algorithm* a) Pick random node. b) Form cluster with it and its attracted neighbors. c) Remove cluster from graph and repeat until nodes remain. *Ailon, Charikar, Newman* [J.ACM 08]

Clustering + Maximal Independent Set

- *Emulating algorithm in two passes:*
  - *Preprocess* Randomly order nodes, $v_1, v_2, \ldots, v_n$.
  - *First Pass* Store all attractive edges incident to $\{v_1, \ldots, v_{\sqrt{n}}\}$. Now can emulate first $\sqrt{n}$ iterations of the algorithm.
  - *Second Pass* Store all remaining attractive edges. Now can emulate remaining steps of the algorithm.
- *Thm* Algorithm uses $\tilde{O}(n^{1.5})$ space. *Ahn et al.* [ICML 16]
  - *Proof Idea* At most $n^{1.5}$ edges stored in first pass. In second, pass, can show remaining node have at most $n^{0.5}$ neighbors.
- With more work, can get $\tilde{O}(n)$ space with $O(\log \log n)$ passes. Can also find maximal independent sets.

Coloring

- *Coloring* With min number of colors, assign a color to every node such that no edge has monochromatic endpoints.



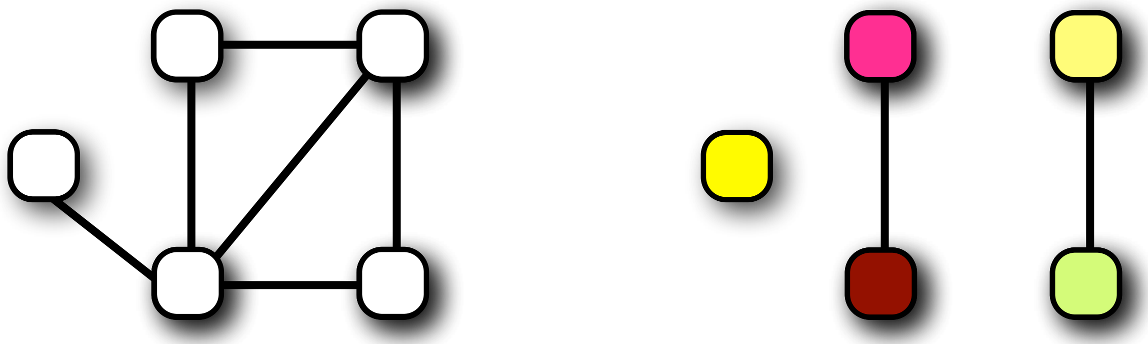
- *Thm* Can color a graph in Δ+1 colors where Δ is max degree.

? How can we do this in a few passes with $\tilde{O}(n)$ space?

- O(Δ log log n) passes via independent sets. Let's do better!

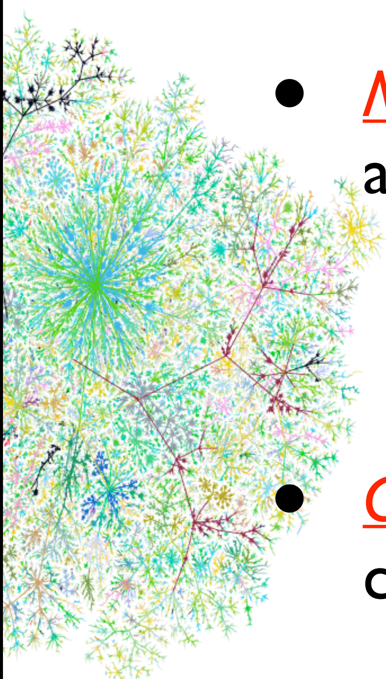
Coloring

- *(1+ε)Δ Coloring* a) Randomly color with Δ/r colors. b) Store edges E' with monochromatic endpoints. c) Shade colors such that E' edges no longer monochromatic. *Bera, Ghosh* [ArXiv 18]



- *Space Analysis* $|E'|=O(nr)$ since probability edge in E' is r/Δ.
- *Colors Analysis* If $r \approx \varepsilon^{-2} \log n$, max degree in E' is $\Delta_{E'} < (1+\varepsilon)r$ and final number of colors is $(1+\Delta_{E'})\Delta/r = (1+\varepsilon)\Delta$.
- *Δ+1 Coloring Idea* For node v, pick $S_v \subset \{1,\ldots,\Delta+1\}$ of $O(\log n)$ random colors. May assume v's color in S_v. *Assadi et al.* [SODA 19]

Wrapping Up



- *Matching Story* Using sketches to compute exact matchings, approximate matchings, and planar matchings.

  "Sketches enable interesting types of sampling via sketches that are useful for graph problems."

- *Connectivity Story* Using sketches to analyze edge and node connectivity, build cut and spectral sparsifiers etc.

  "Homomorphic compression: sketch first, compute later."

- *Other Stories* Densest subgraph, clustering coefficients and triangles, correlation clustering, set cover and hitting sets…

Thanks!