Automatic Round Elimination:

A New Approach for Proving Complexity Bounds in the LOCAL Model

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ETH Zurich

joint work with: Alkida Balliu, Orr Fischer, Juho Hirvonen, Barbara Keller, Tuomo Lempiäinen, Dennis Olivetti, Mikaël Rabie, Joel Rybicki, Jukka Suomela, Jara Uitto

Lovász Local Lemma

There is no randomized LLL algorithm with complexity $o(\log \log n)$.

[B., Fischer, Hirvonen, Keller, Lempiäinen, Rybicki, Suomela, Uitto, STOC'16]

There is no deterministic LLL algorithm with complexity $o(\log n)$.

[Chang, Kopelowitz, Pettie, FOCS'16]

... Automatic Round Elimination ... [B., PODC'19]

Maximal Matching

There is no randomized MM algorithm with complexity

$$o(\Delta) + o\left(\frac{\log\log n}{\log\log\log n}\right).$$

There is no deterministic MM algorithm with complexity

$$o(\Delta) + o\left(\frac{\log n}{\log\log n}\right).$$

[Balliu, B., Hirvonen, Olivetti, Rabie, Suomela, FOCS'19]

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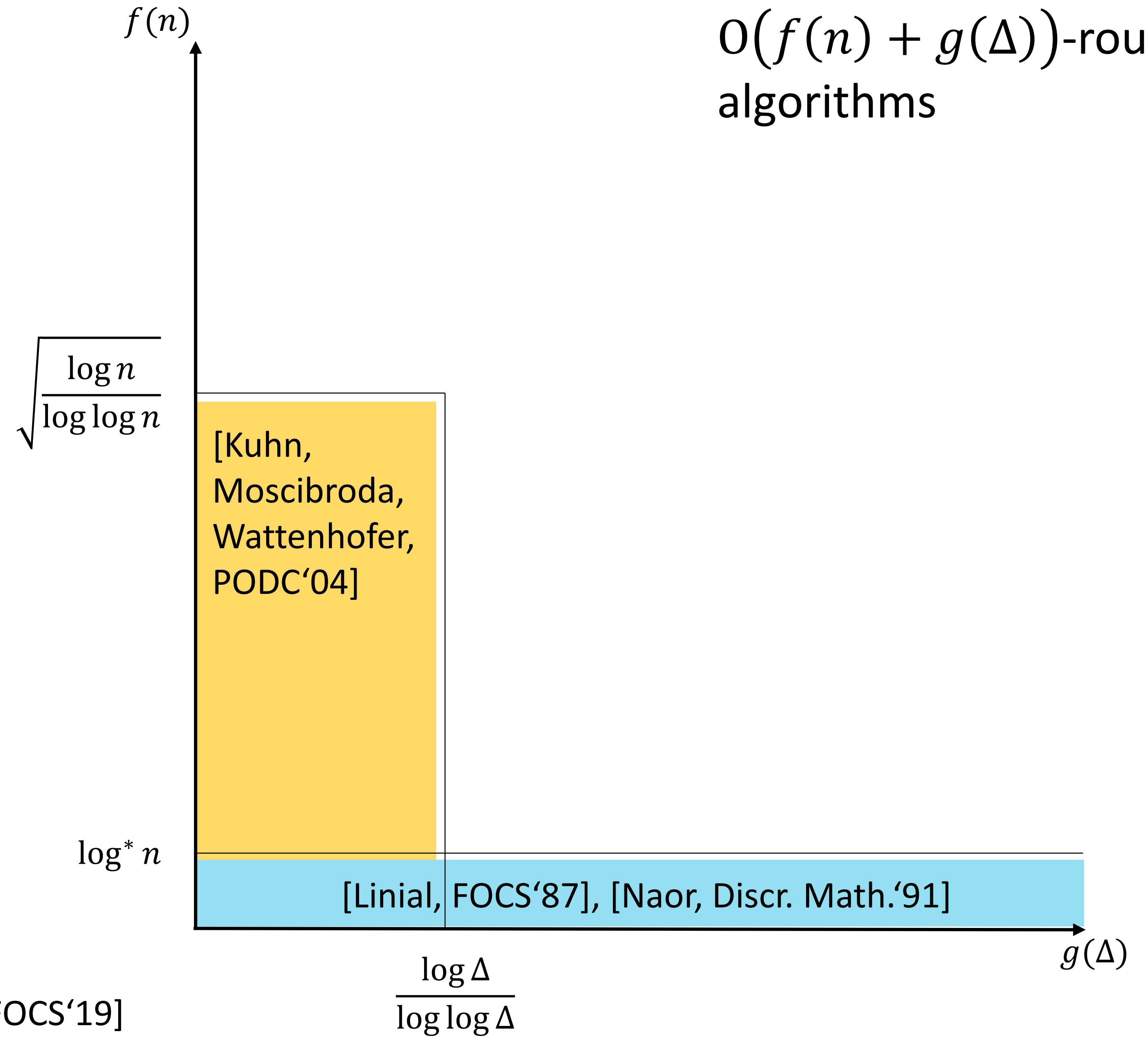
[Balliu, B., Hirvonen, Olivetti, Rabie, Suomela, FOCS‘19]

$O(f(n) + g(\Delta))$-round algorithms



Maximal Matching

$O(f(n) + g(\Delta))$-round randomized algorithms

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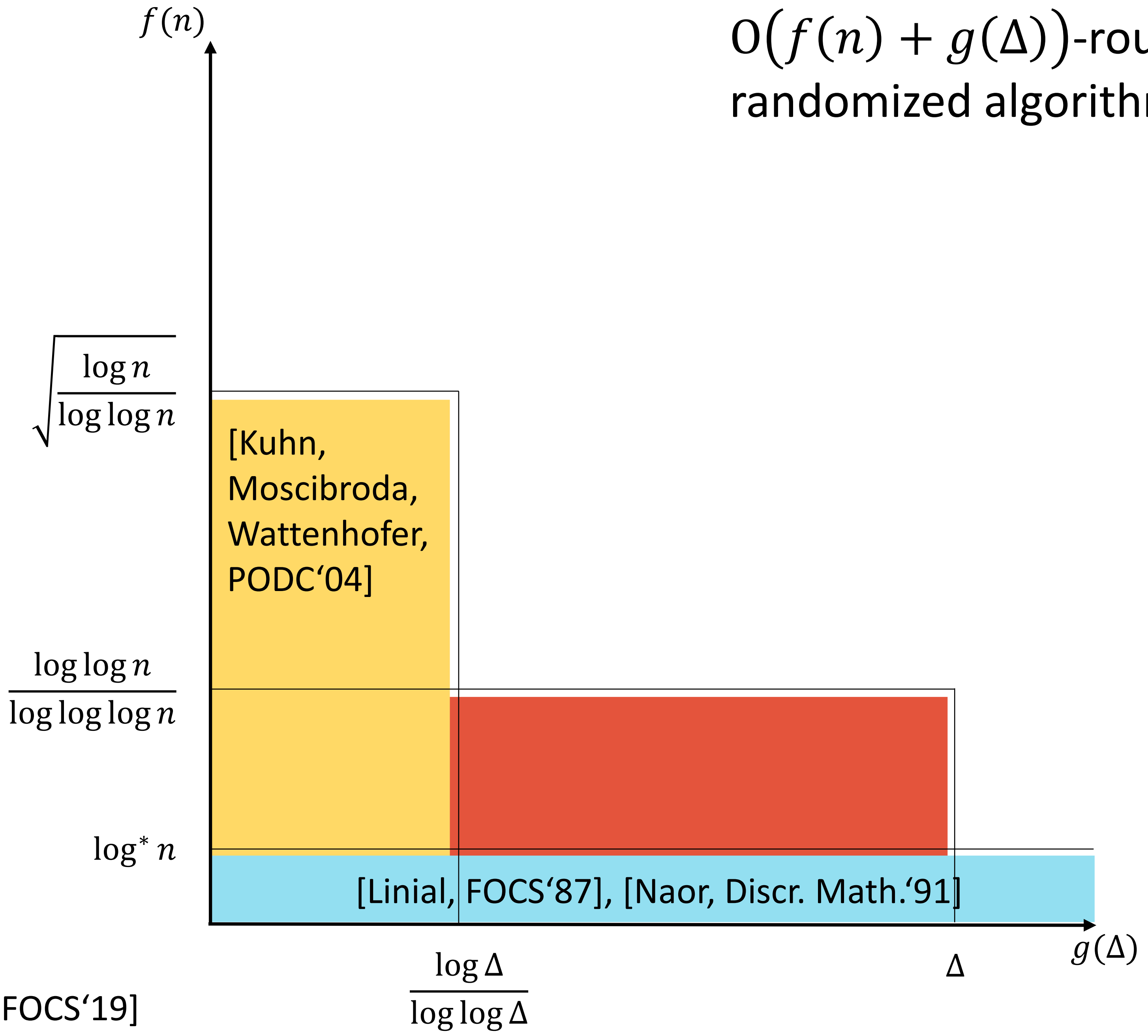

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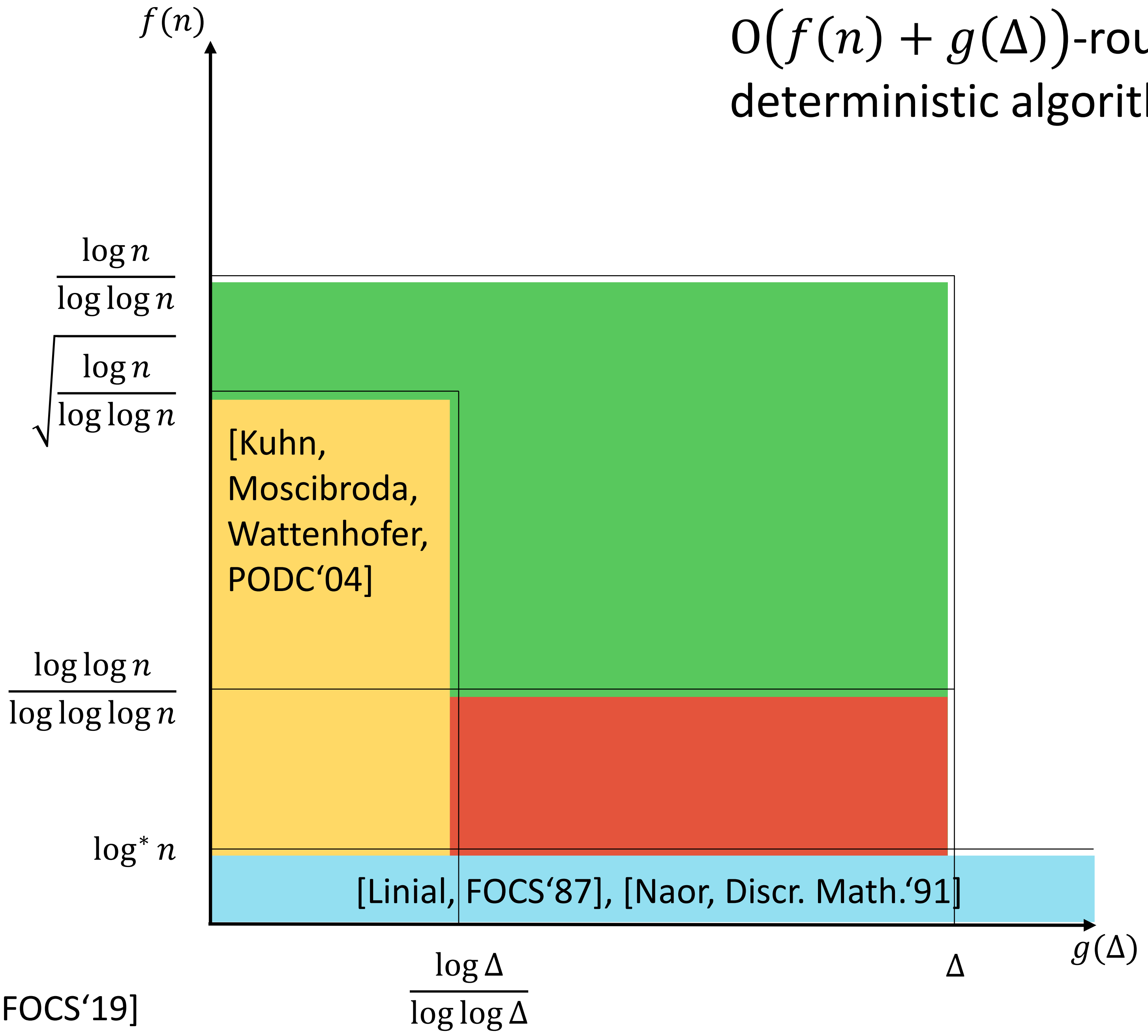
$O(f(n) + g(\Delta))$-round deterministic algorithms



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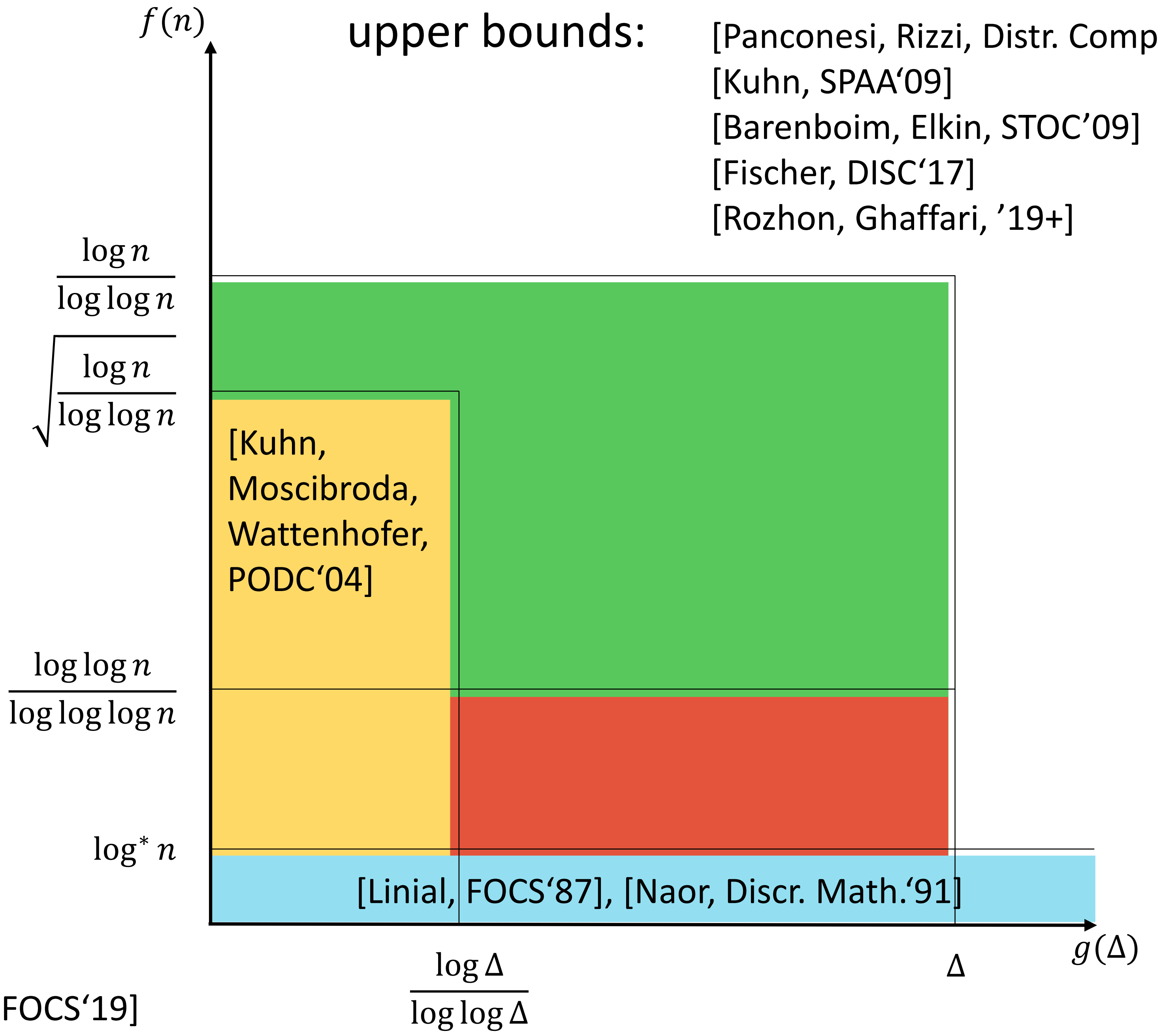
[Balliu, B., Hirvonen, Olivetti, Rabie, Suomela, FOCS'19]

upper bounds: [Panconesi, Rizzi, Distr. Comp.'01] [Kuhn, SPAA'09] [Barenboim, Elkin, STOC'09] [Fischer, DISC'17] [Rozhon, Ghaffari, '19+]



The LOCAL Model

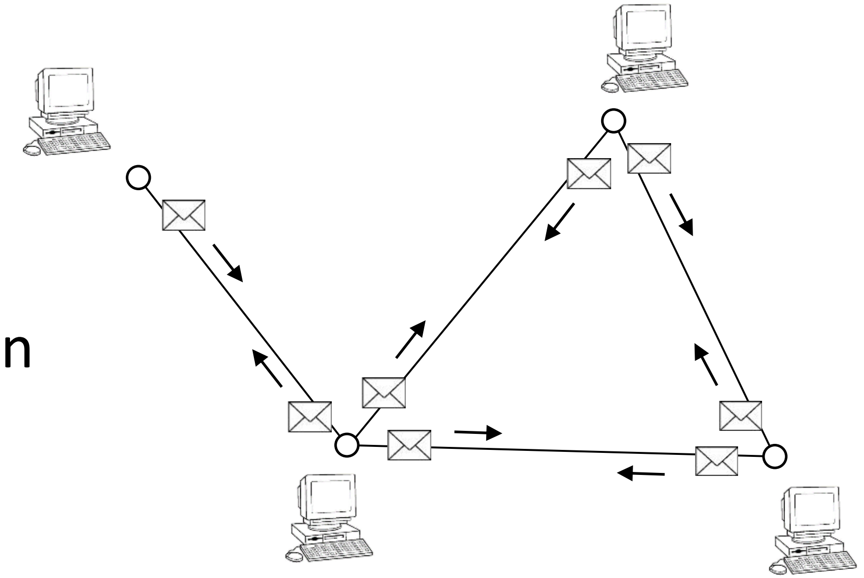
- Synchronous rounds of
  1) Communication
  2) Computation

- Unlimited Message Size and Computation

- Runtime = number of rounds

- $O(\log n)$-bit unique identifiers

[Linial, FOCS'87]



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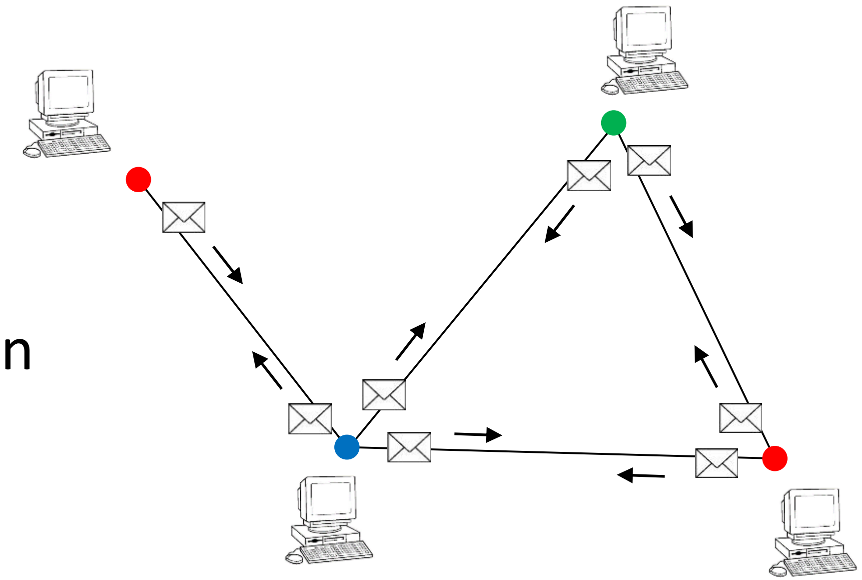

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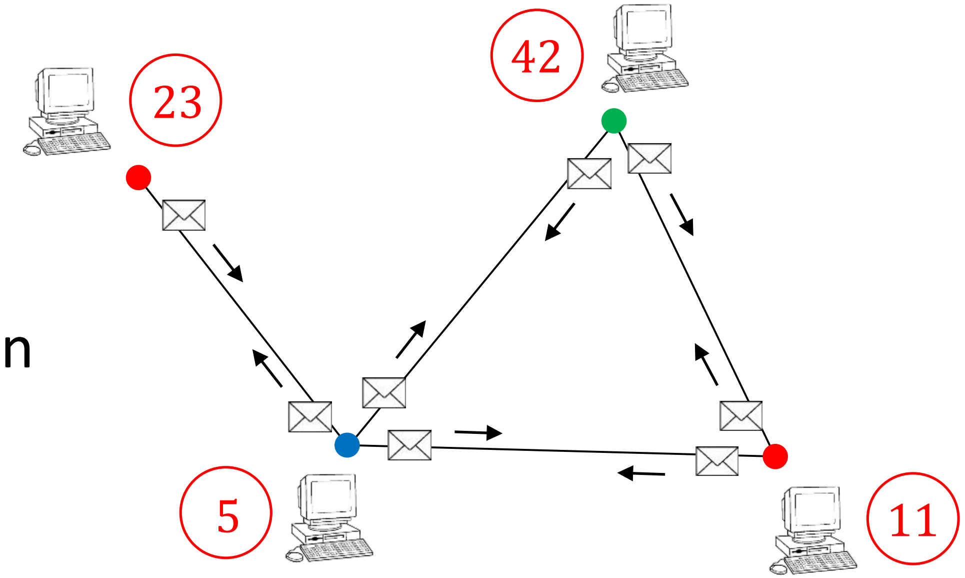

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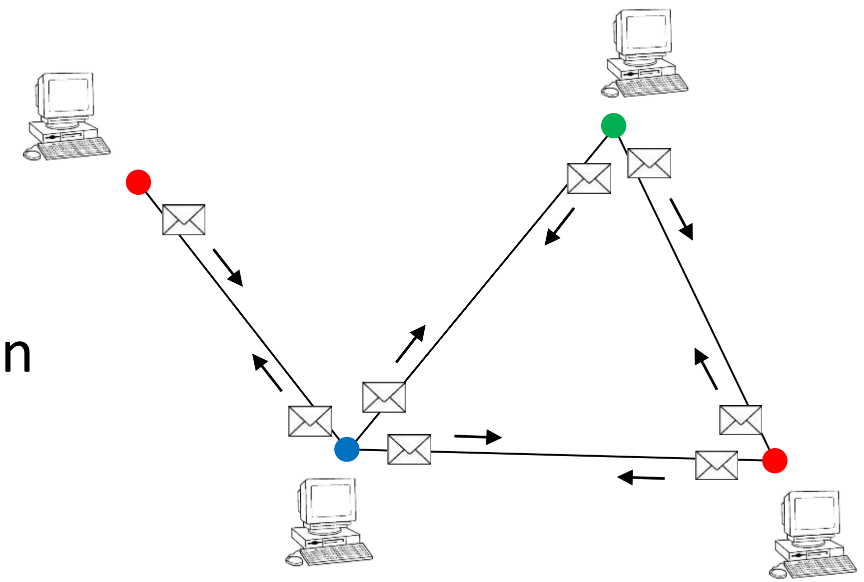

The LOCAL Model

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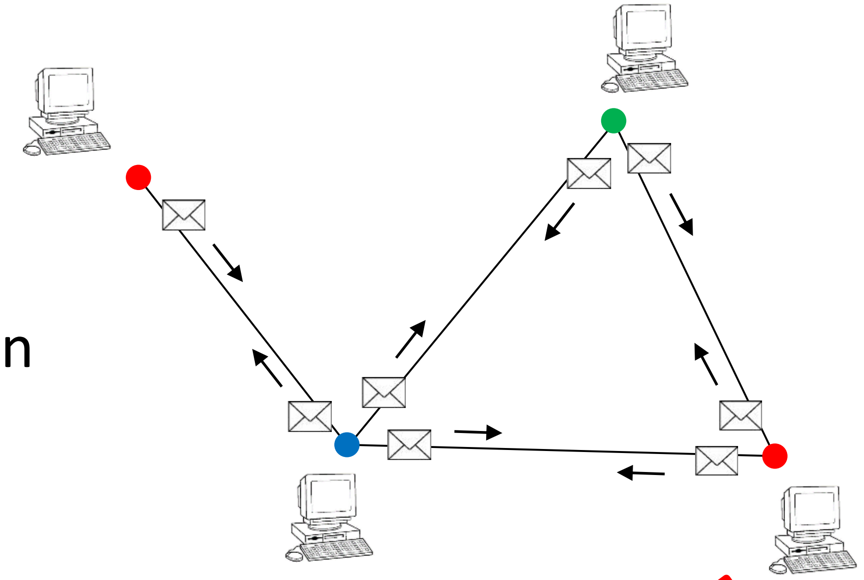


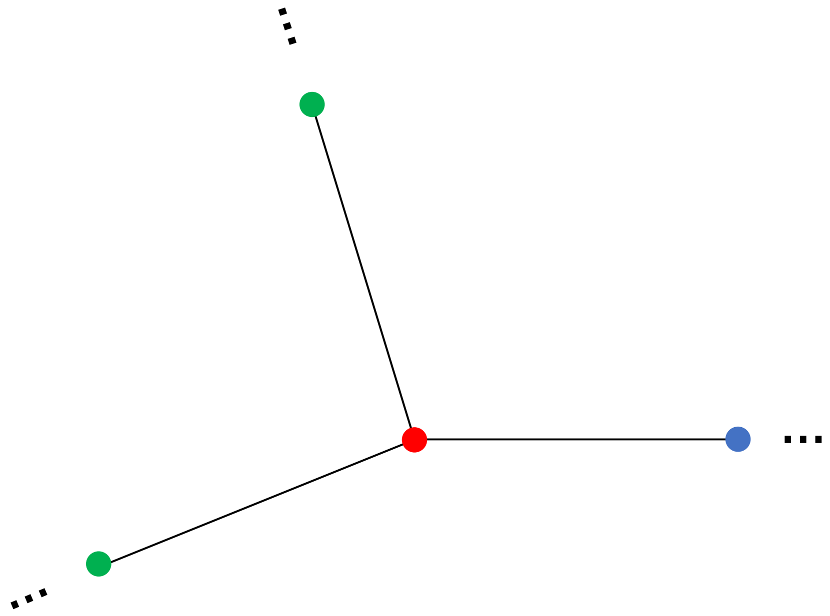
determinstic algorithms

High-girth graphs

Locally Checkable Problems

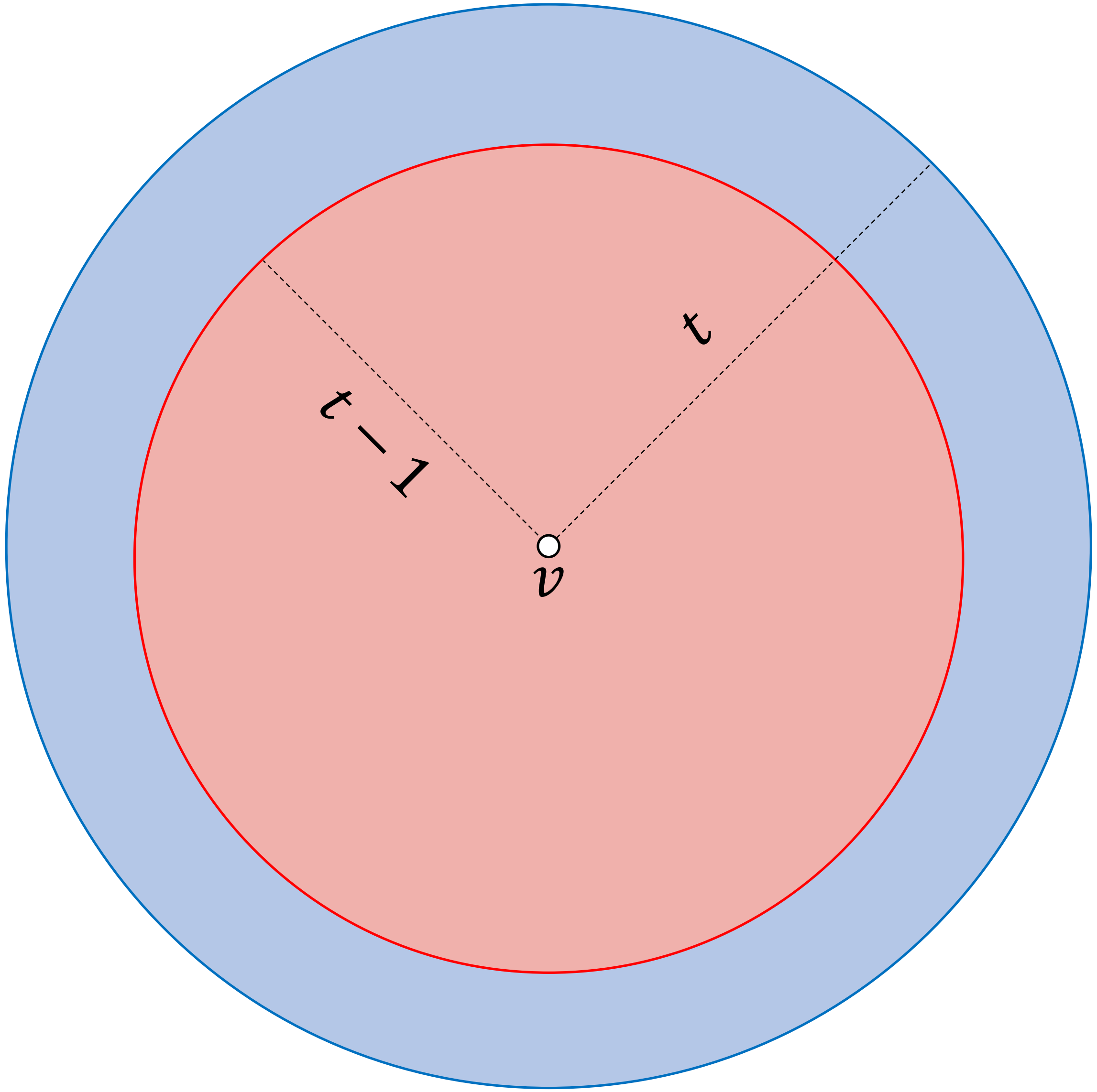
Locally Checkable:

Output correctness is defined via local ($= O(1)$-hop) constraints.



Round Elimination

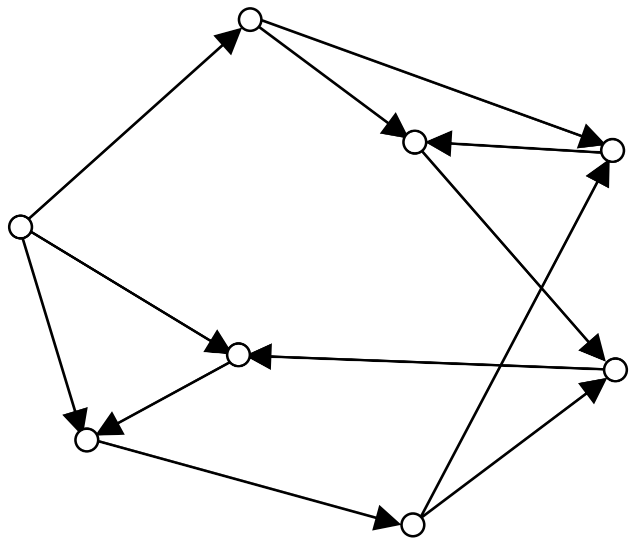
Given that we can solve some problem in t rounds, what can we do in $t-1$ rounds?



Sinkless Orientation

Sinkless Orientation Problem:

Orient the edges such that no node is a sink.



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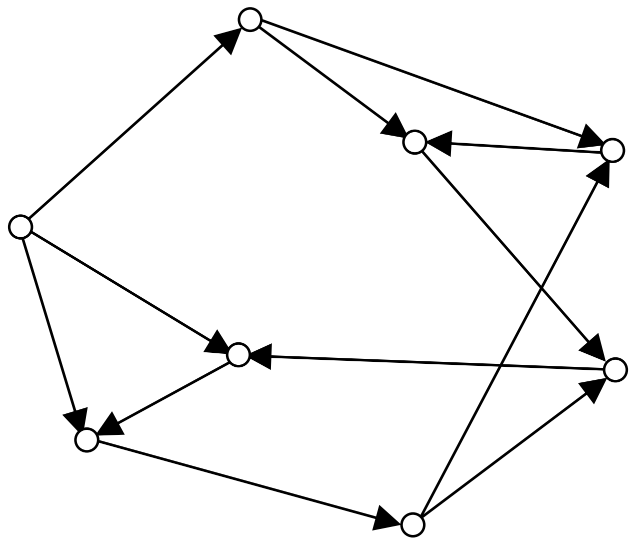
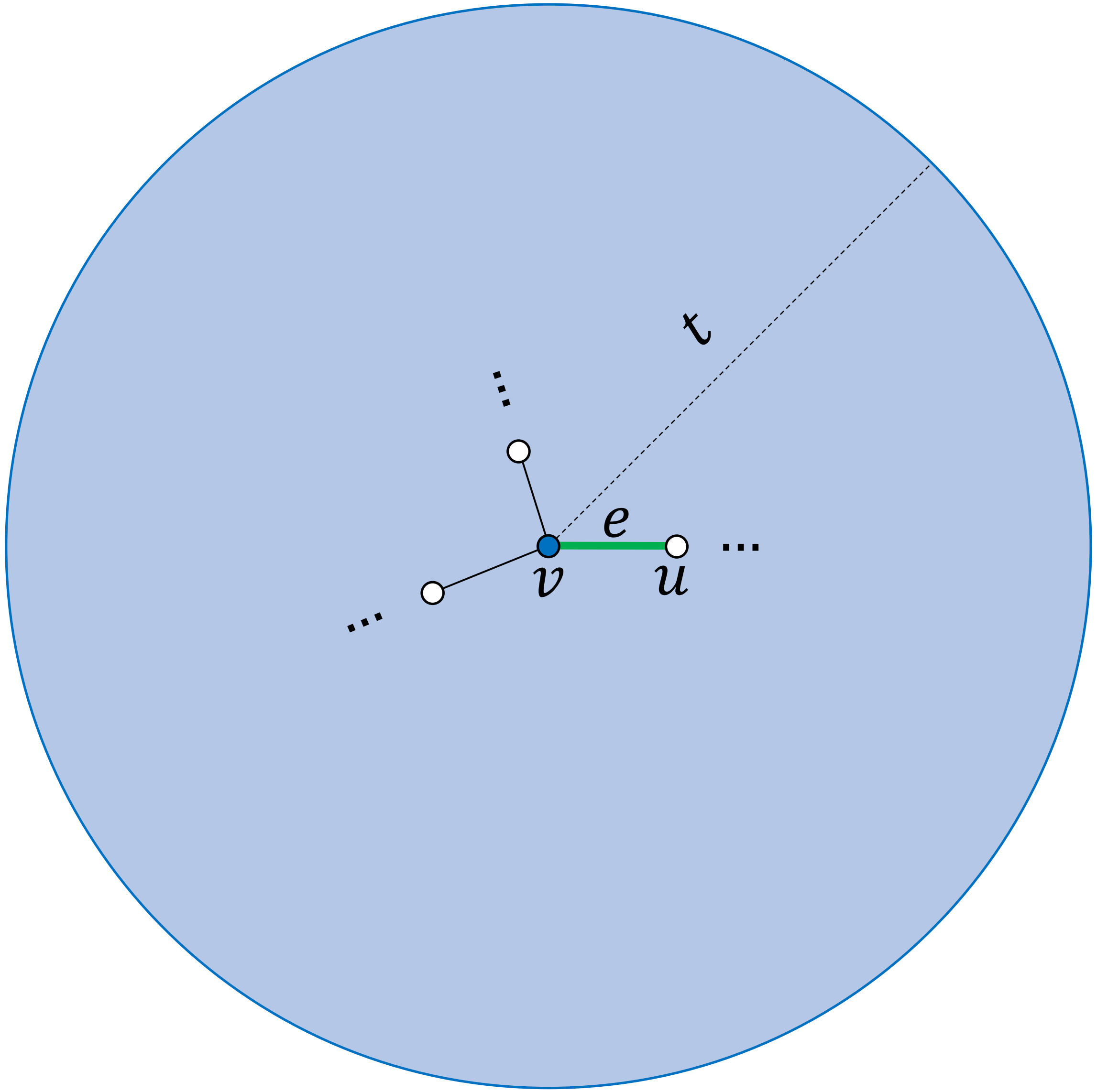


SO is an LLL problem!

Some Intuition



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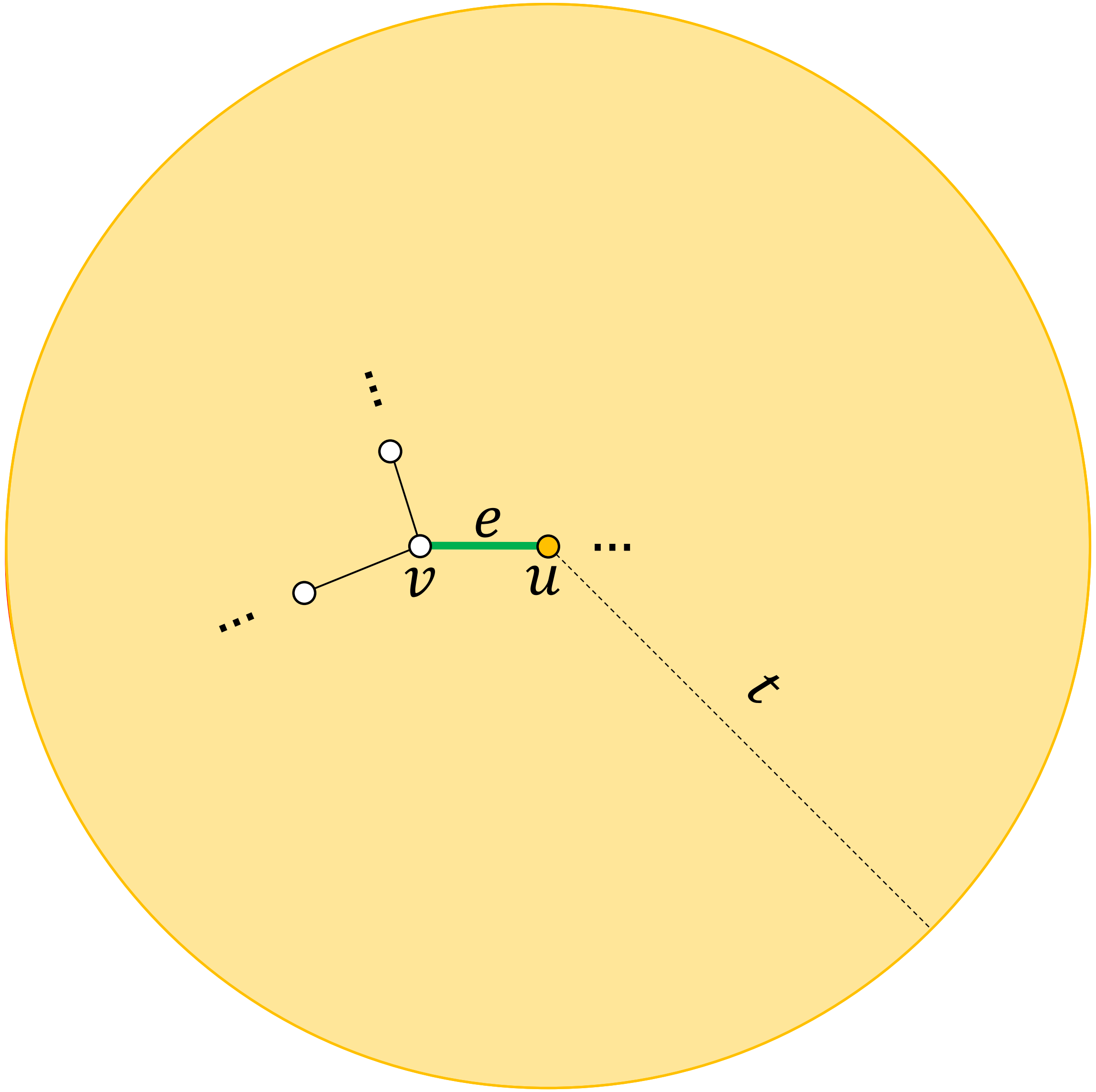

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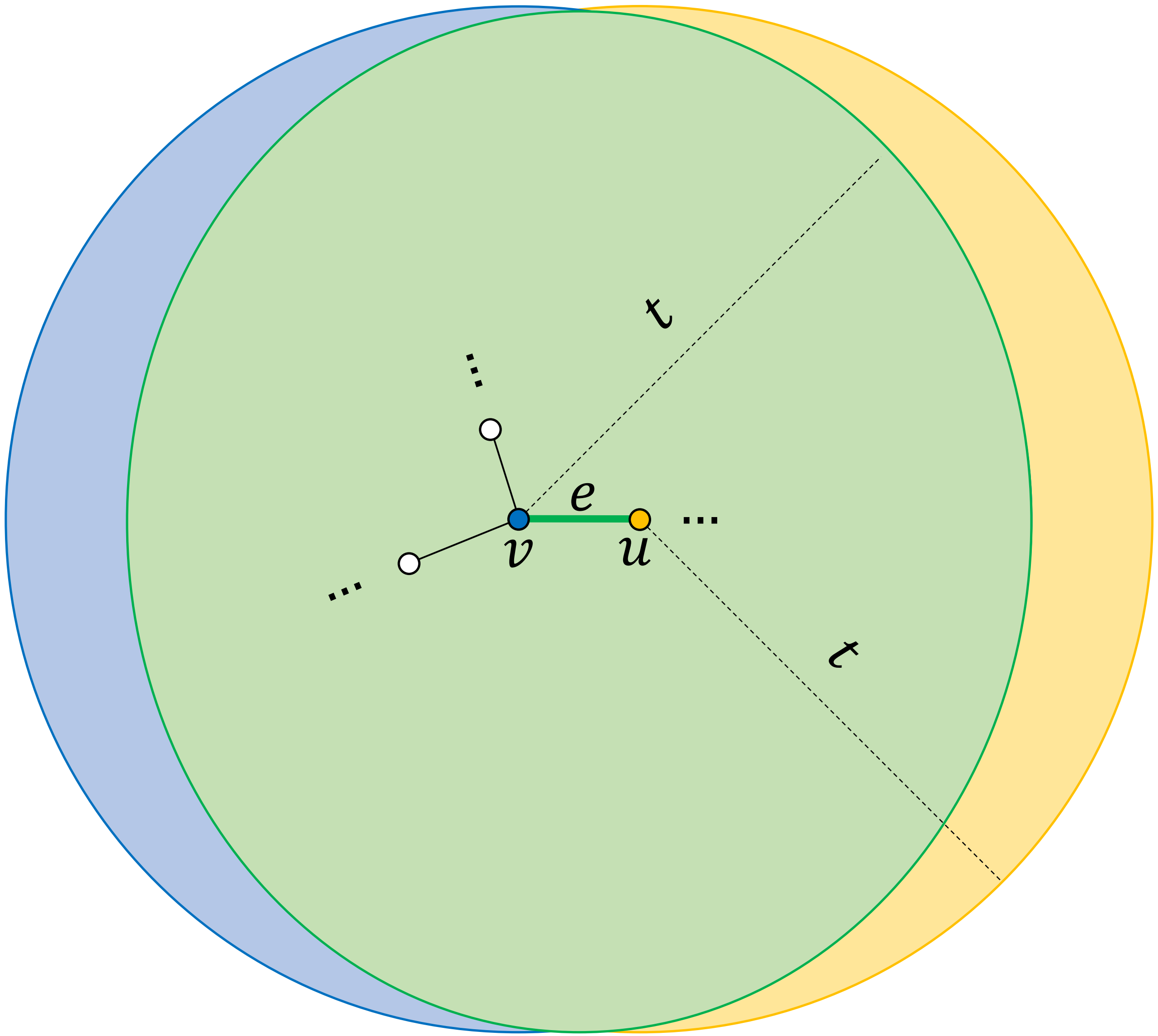

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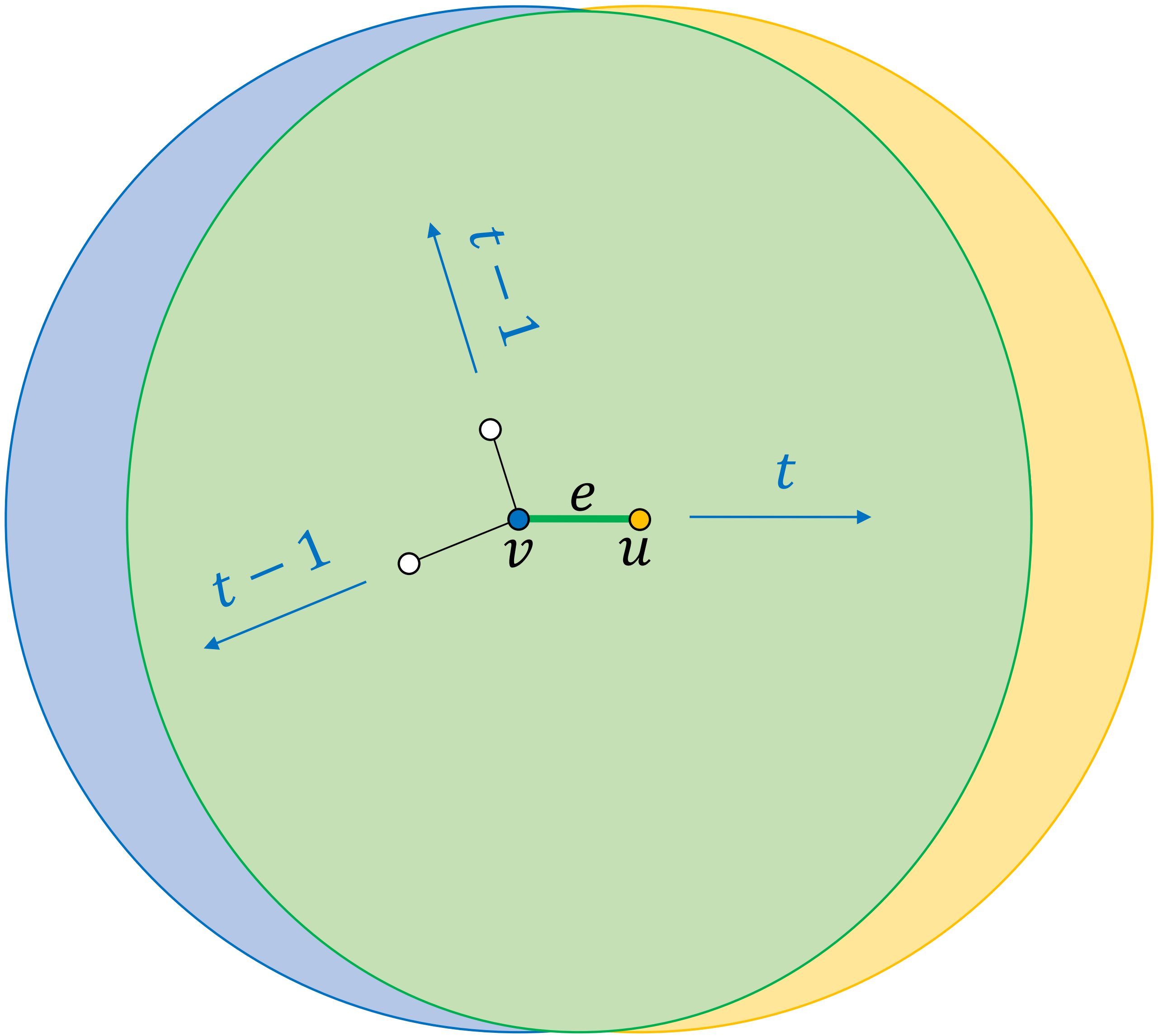

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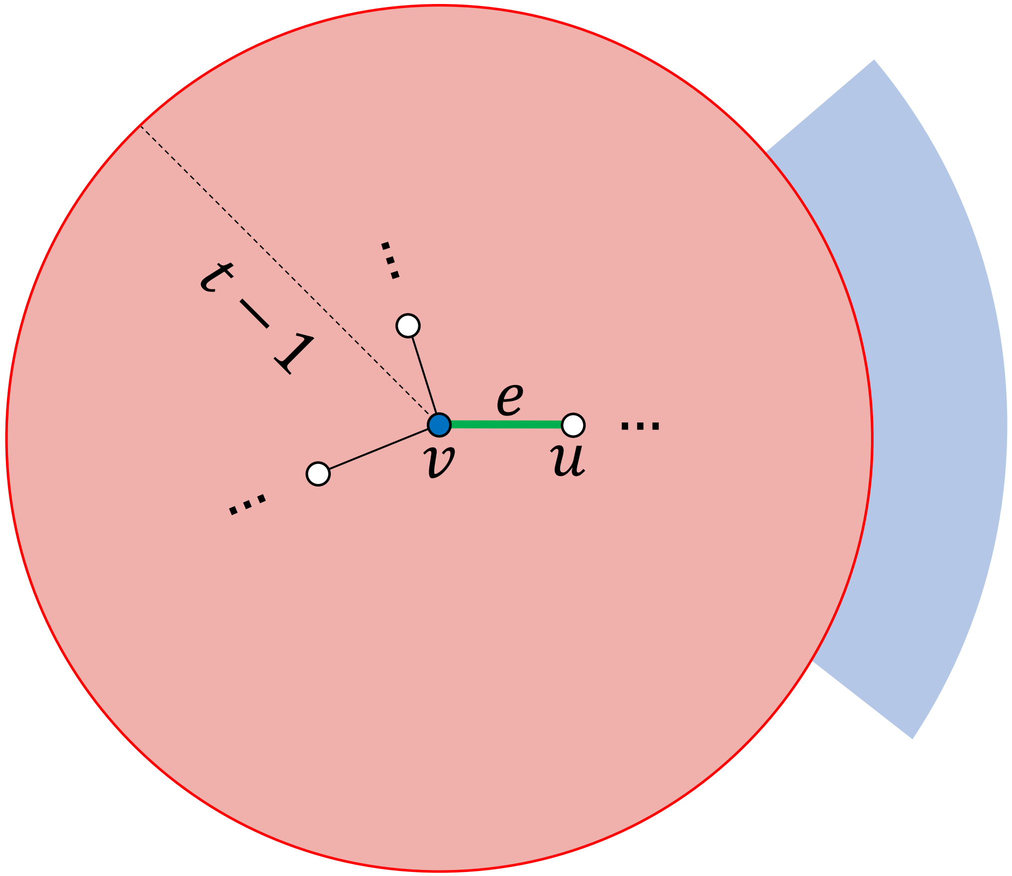

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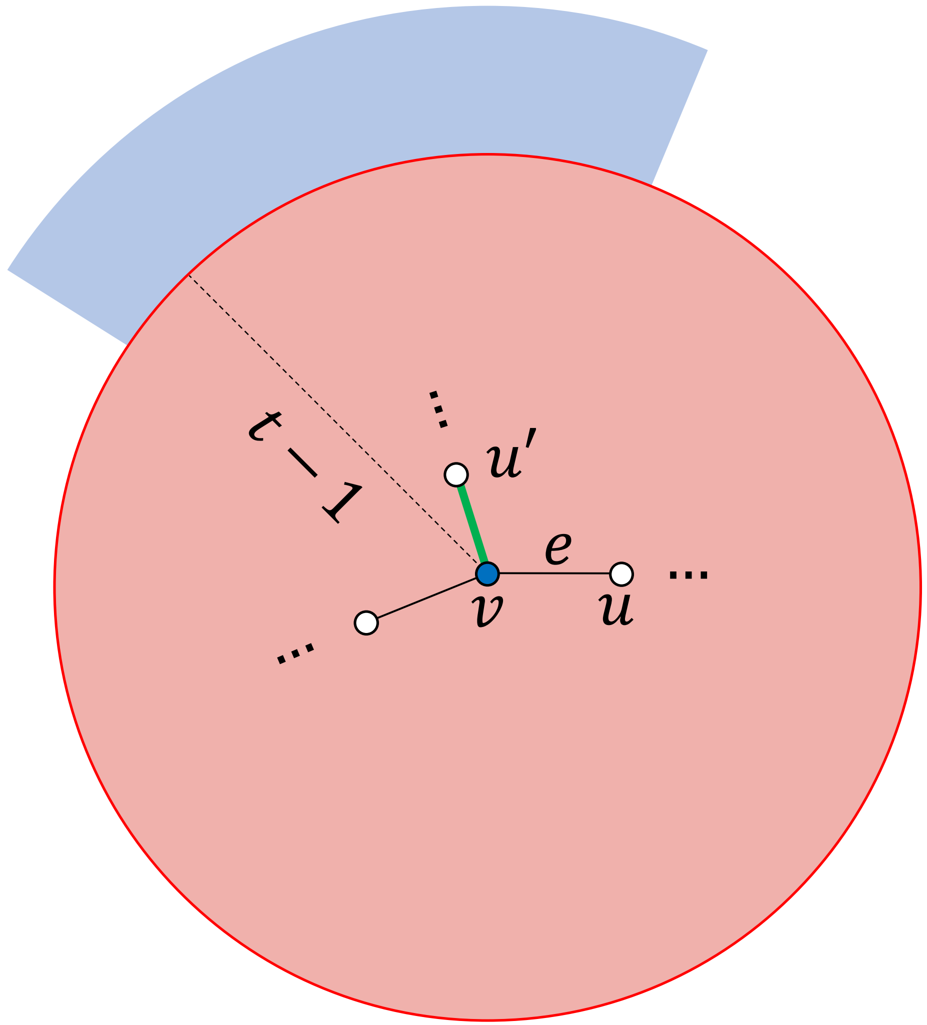

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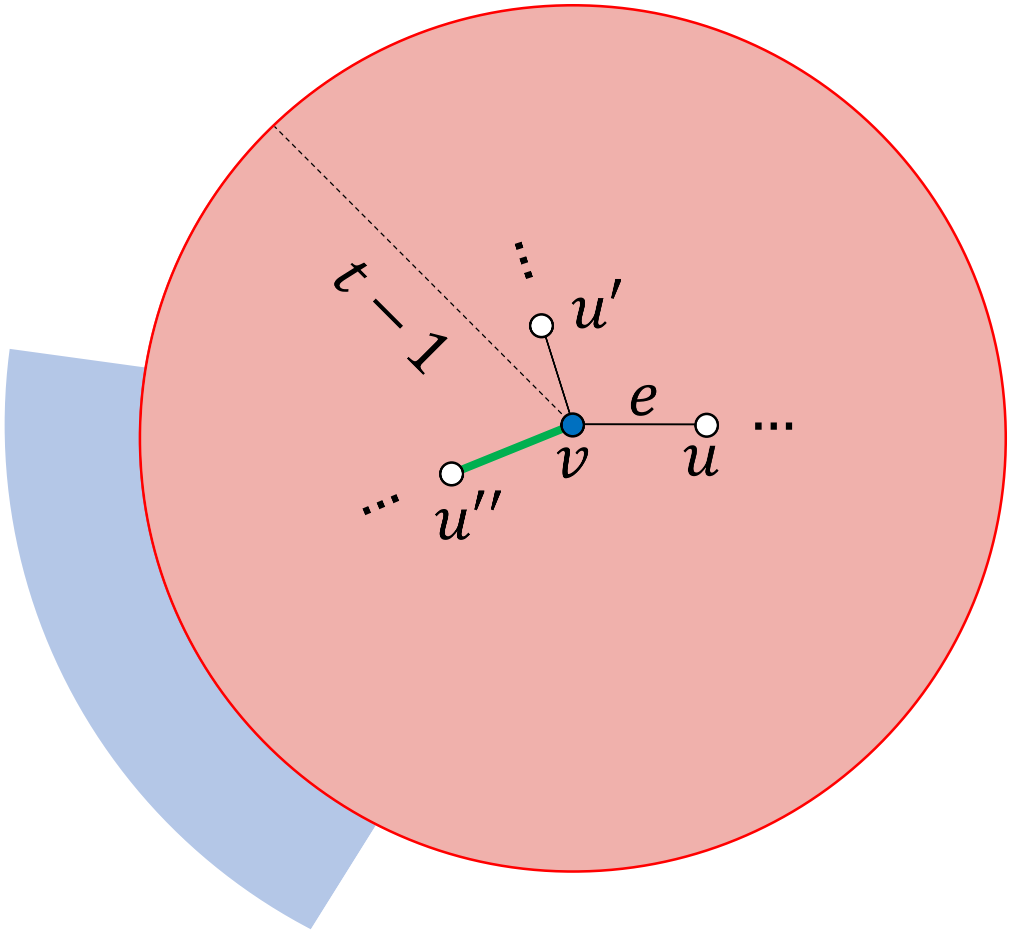

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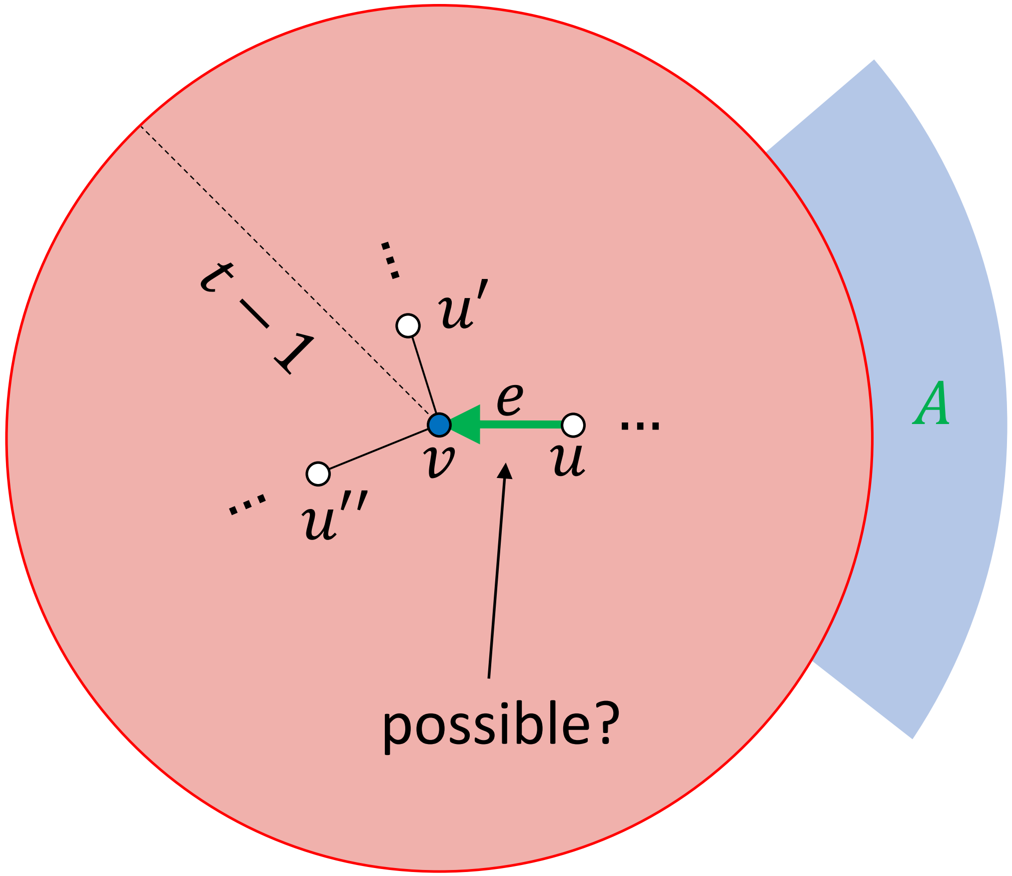

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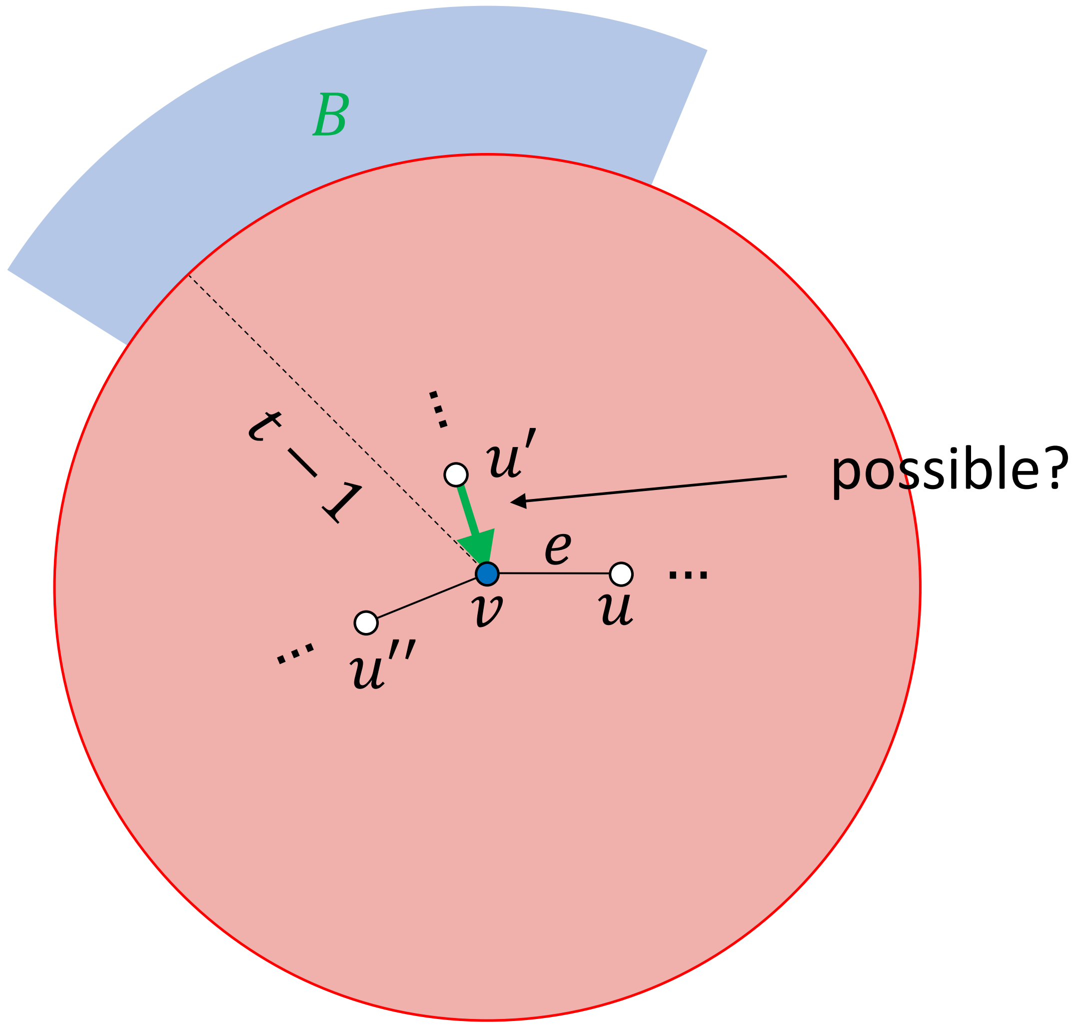

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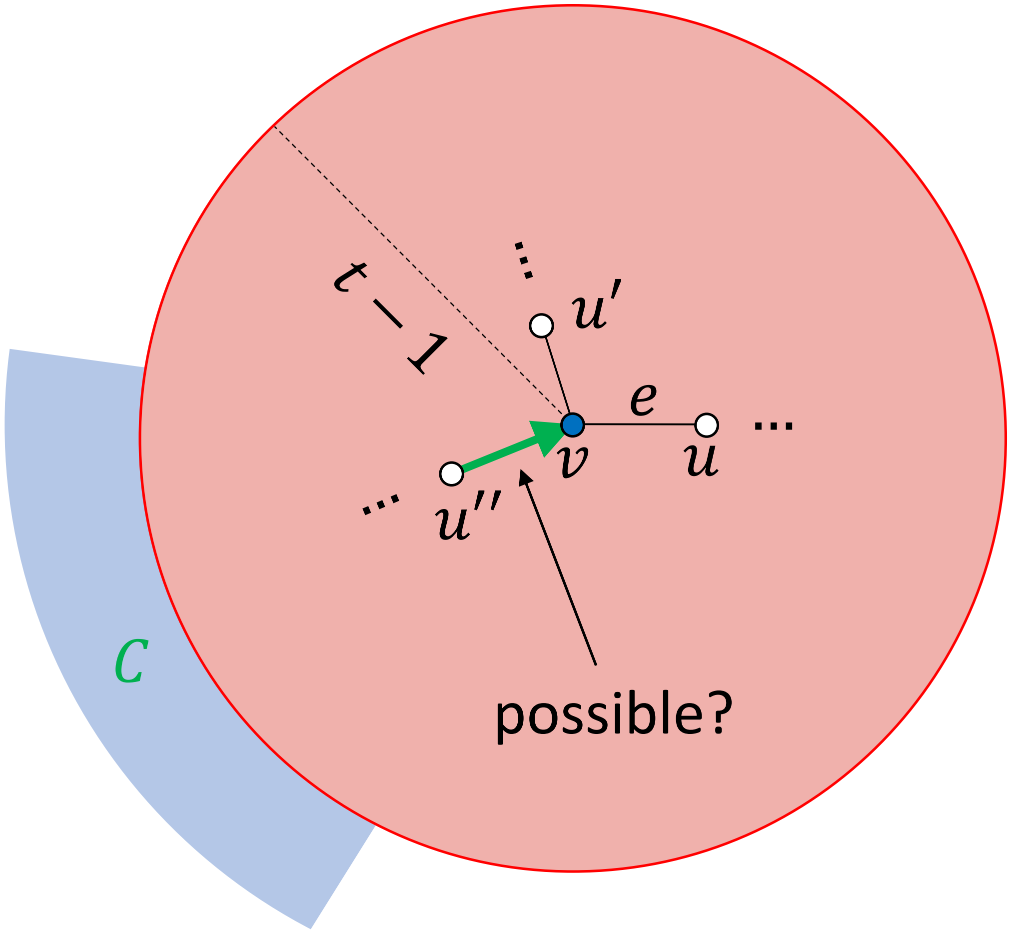

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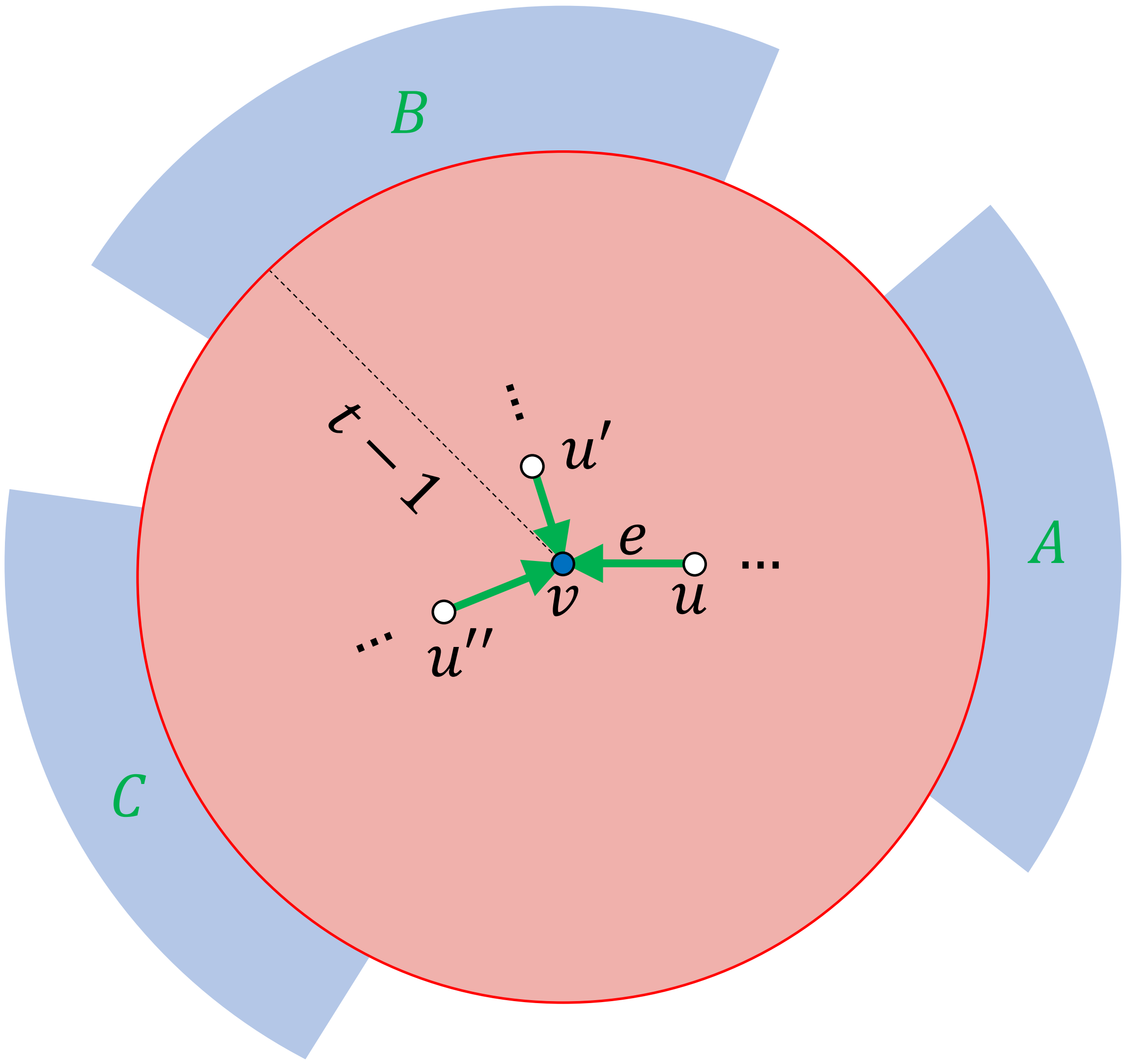
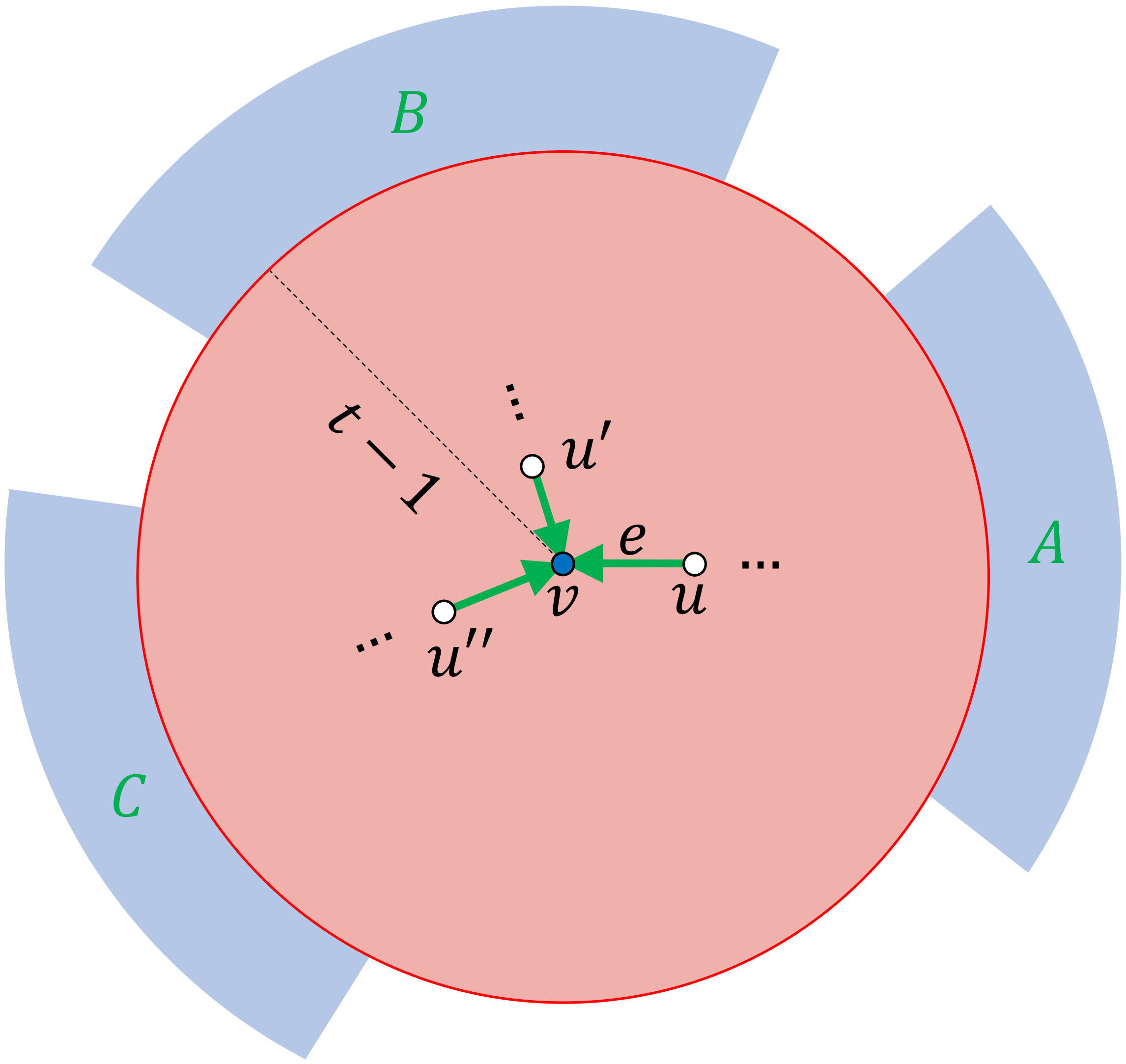


Contradiction!

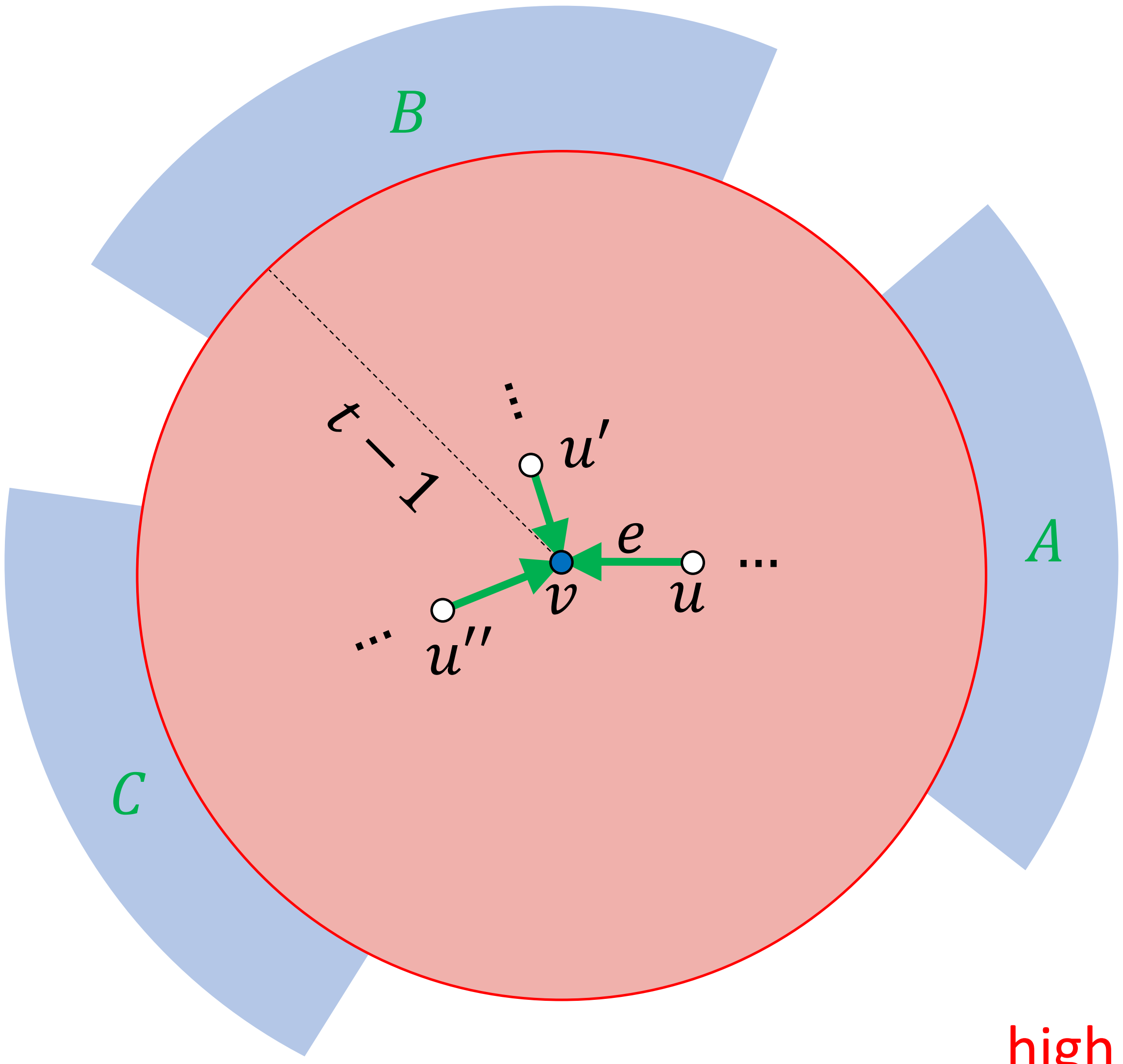
Some Intuition



Contradiction!

(if the extensions are independent!)

Some Intuition



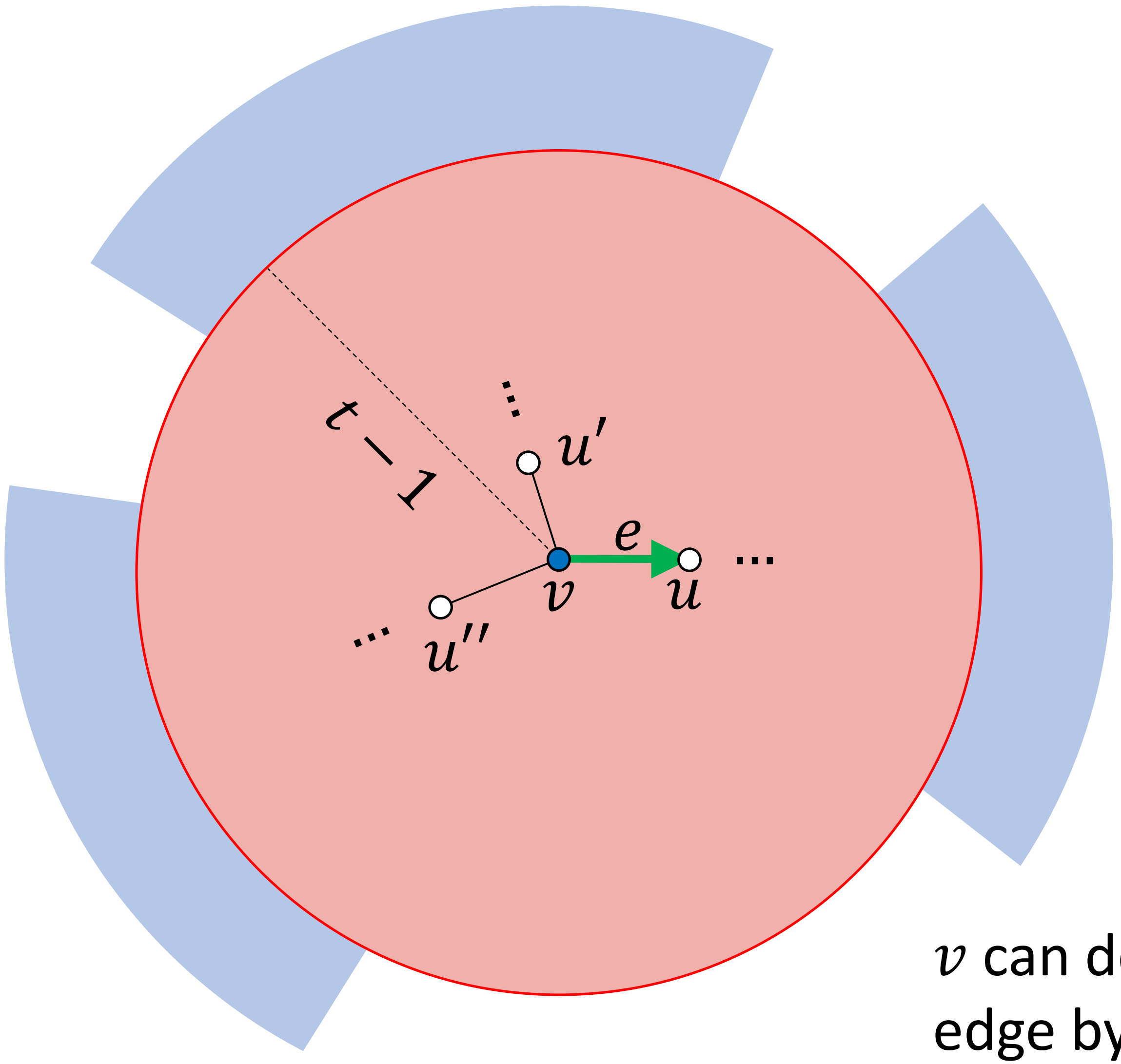
Contradiction!

(if the extensions are independent!)

high girth

no unique IDs

Some Intuition



v can determine one outgoing edge by just looking at its $(t-1)$-hop neighborhood

Round Elimination for SO

t-round algorithm
for sinkless orientation

$\Downarrow$

$(t-1)$-round algorithm
for sinkless orientation

[B., Fischer, Hirvonen, Keller, Lempiäinen, Rybicki, Suomela, Uitto, STOC'16]

Round Elimination for SO

t-round algorithm
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$(t-1)$-round algorithm
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$\Rightarrow$ … $\Rightarrow$

0-round algorithm for
sinkless orientation

[B., Fischer, Hirvonen, Keller, Lempiäinen, Rybicki, Suomela, Uitto, STOC'16]

Round Elimination for SO

t-round algorithm
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$(t-1)$-round algorithm
for sinkless orientation $\Rightarrow$ ... $\Rightarrow$ 0-round algorithm for sinkless orientation

Does not exist!

[B., Fischer, Hirvonen, Keller, Lempiäinen, Rybicki, Suomela, Uitto, STOC'16]

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Round Elimination for SO

Does not exist!

t-round algorithm
for sinkless orientation

$t \in \Theta(\log n)$

$\Downarrow$

$(t-1)$-round algorithm
for sinkless orientation $\Rightarrow$ ... $\Rightarrow$ 0-round algorithm for
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Does not exist!

[B., Fischer, Hirvonen, Keller, Lempiäinen, Rybicki, Suomela, Uitto, STOC'16]

Round Elimination for SO

t-round algorithm
for sinkless orientation

$\Downarrow$

$(t-1)$-round algorithm
for sinkless orientation

What about
other problems?

Round Elimination for Coloring Rings

t-round algorithm
for sinkless orientation

$\Downarrow$

$(t-1)$-round algorithm
for sinkless orientation

What about other problems?

t-round algorithm
for k-coloring rings

$\Downarrow$

$(t-1)$-round algorithm
for 2^k-coloring rings

[Linial, FOCS'87]

Other Problems?

t-round algorithm
for k-coloring rings

$\Downarrow$

$(t-1)$-round algorithm
for 2^k-coloring rings

t-round algorithm
for sinkless orientation

$\Downarrow$

$(t-1)$-round algorithm
for sinkless orientation

What about
other problems?

Even-Degree Weak 2-Coloring

[Balliu, Hirvonen, Olivetti,
Suomela, PODC'19]

Round Elimination for SO

t-round algorithm
for sinkless orientation

$$\Downarrow$$

$(t-1)$-round algorithm
for sinkless orientation

Automatic Round Elimination

Let Π_0 be any locally checkable problem. Then we can automatically find a locally checkable problem Π_1 such that

t-round algorithm for sinkless orientation

$\Downarrow$

$(t-1)$-round algorithm for sinkless orientation

t-round algorithm for Π_0

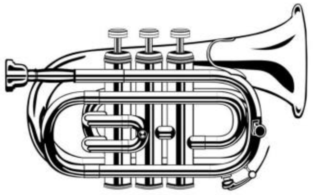
$\Updownarrow$

$(t-1)$-round algorithm for Π_1

[B., PODC'19]

Automatic Round Elimination

Let Π_0 be any locally checkable problem. Then we can automatically find a locally checkable problem Π_1 such that

Π_0  Π_1

t-round algorithm for Π_0

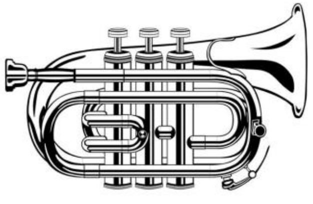
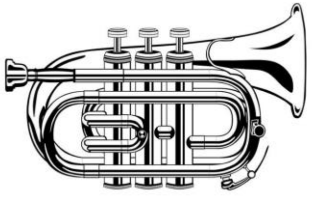
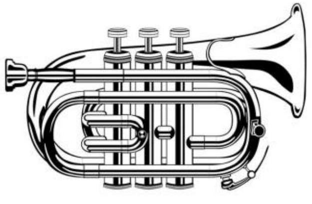
$\Updownarrow$

$(t-1)$-round algorithm for Π_1

Problem	Π_0	Π_1
Complexity	$T(n, \Delta)$	$T(n, \Delta) - 1$

[B., PODC'19]

Automatic Round Elimination

Π_0  Π_1  Π_2  ...

Problem	Π_0	Π_1	Π_2	...
Complexity	$T(n, \Delta)$	$T(n, \Delta) - 1$	$T(n, \Delta) - 2$	...

Obtaining Complexities

Find the first problem in the sequence that can be solved in 0 rounds …

Problem	Π_0	Π_1	Π_2	…	Π_k
Complexity	$T(n, \Delta)$	$T(n, \Delta) - 1$	$T(n, \Delta) - 2$	…	0

Obtaining Complexities

Find the first problem in the sequence that can be solved in 0 rounds …

Problem	Π_0	Π_1	Π_2	…	Π_k
Complexity	$T(n,\Delta)$	$T(n,\Delta)-1$	$T(n,\Delta)-2$	…	0

Π_0 has complexity k.

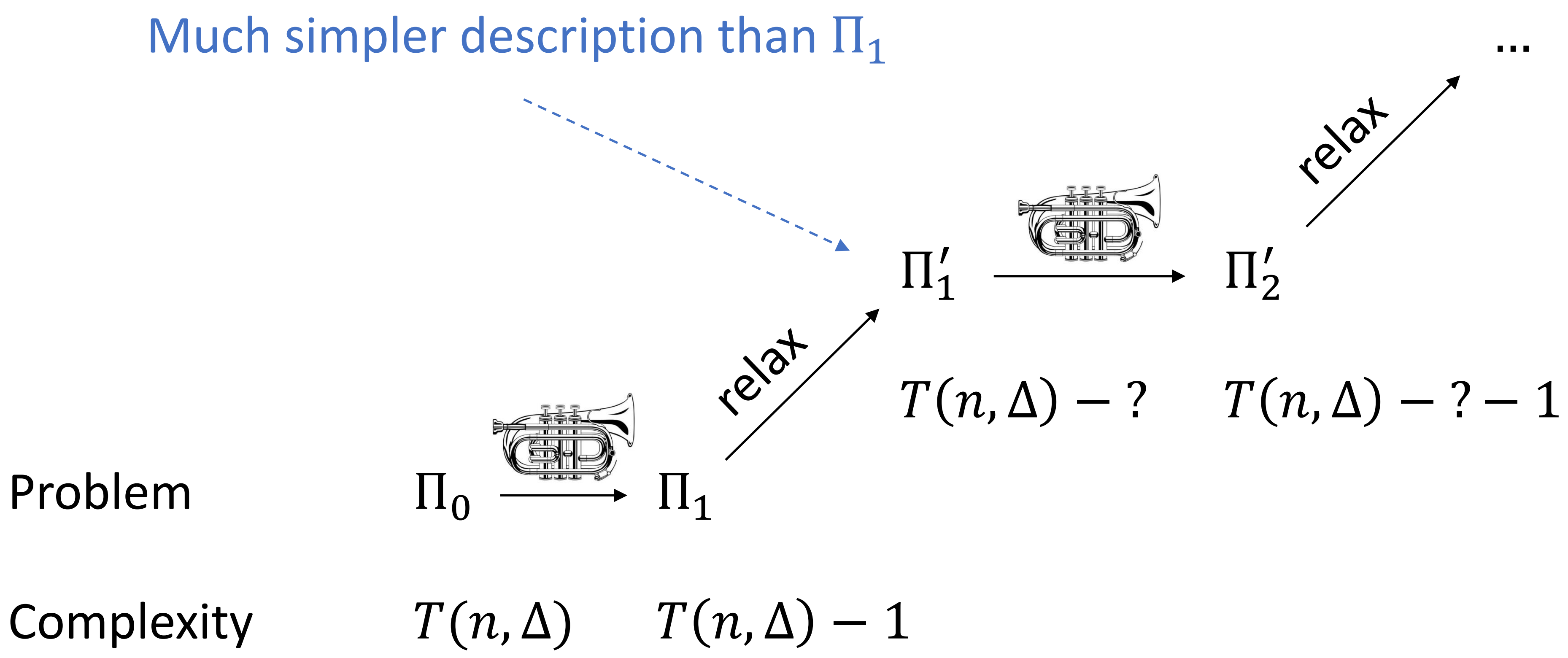
Where’s the catch?

Increasingly complex problem descriptions!

Problem	Π_0	Π_1	Π_2	…	Π_k
Complexity	$T(n, \Delta)$	$T(n, \Delta) - 1$	$T(n, \Delta) - 2$	…	0

Π_0 has complexity k.

Simplifying the Problems



Lower Bounds

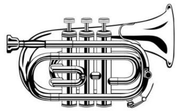
Π_k^*

0

Much simpler description than Π_1

...

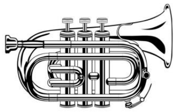
relax



Π_1' → Π_2'

$T(n, \Delta) - ?$ $\quad T(n, \Delta) - ? - 1$

relax



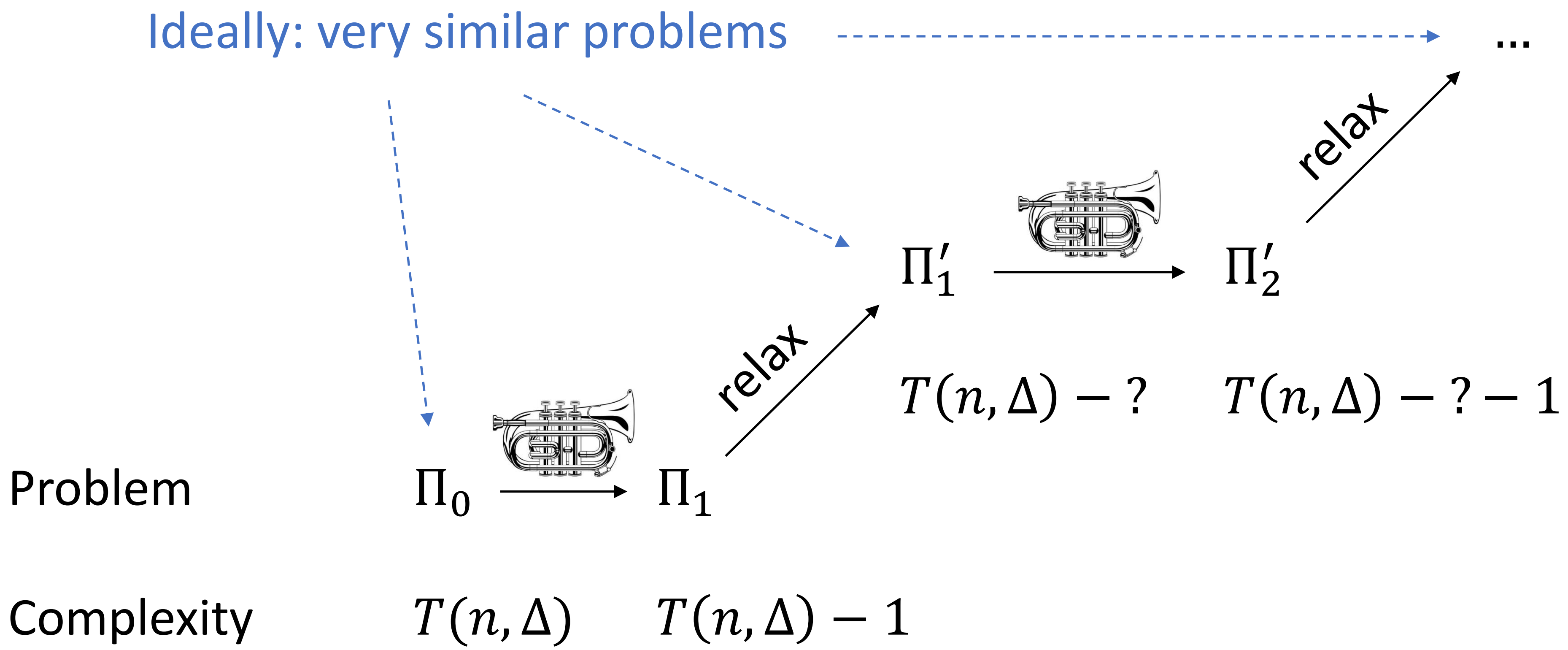
Problem	Π_0 →	Π_1
Complexity	$T(n, \Delta)$	$T(n, \Delta) - 1$

Π_0 has complexity at least k.

Lower Bounds

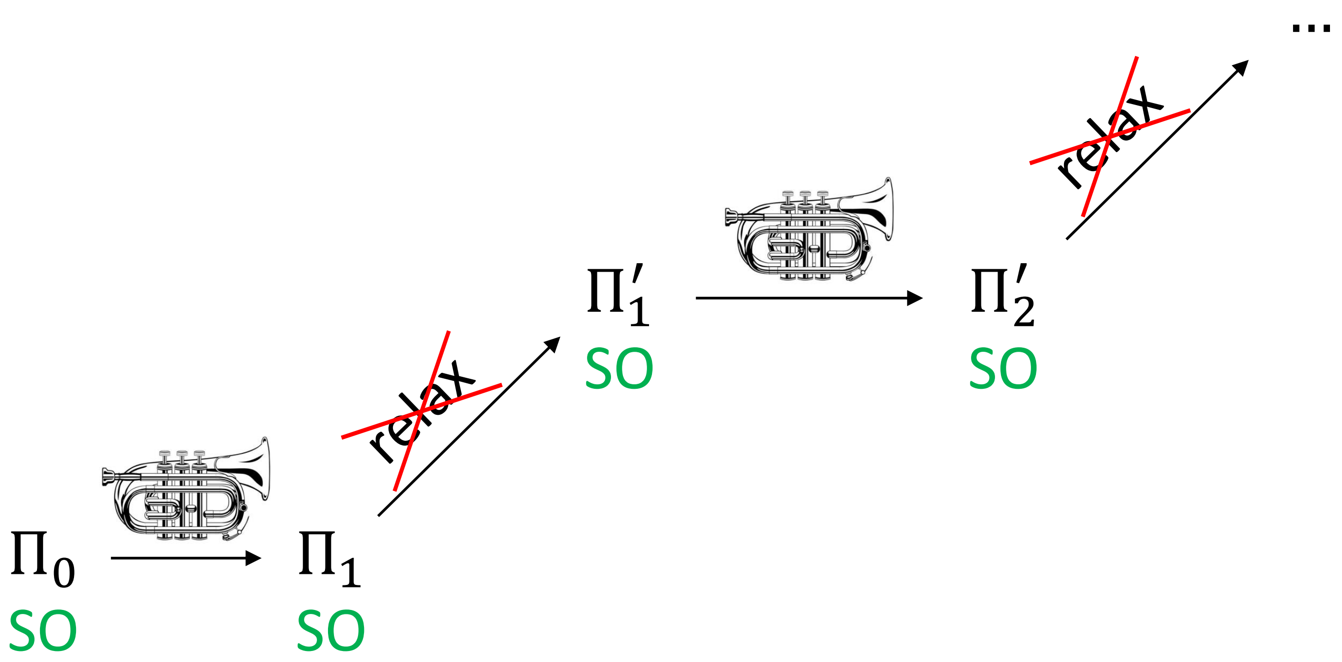
Π_k^*

0

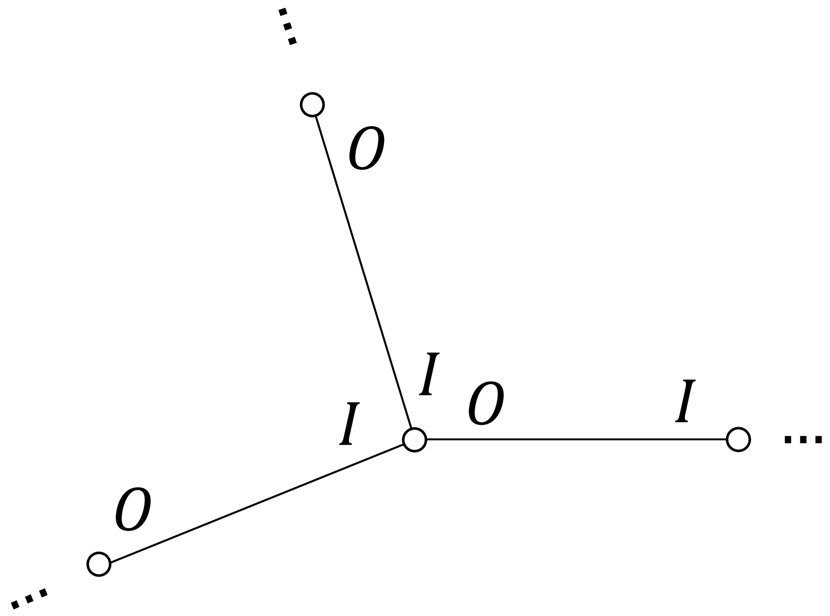


Π_0 has complexity at least k.

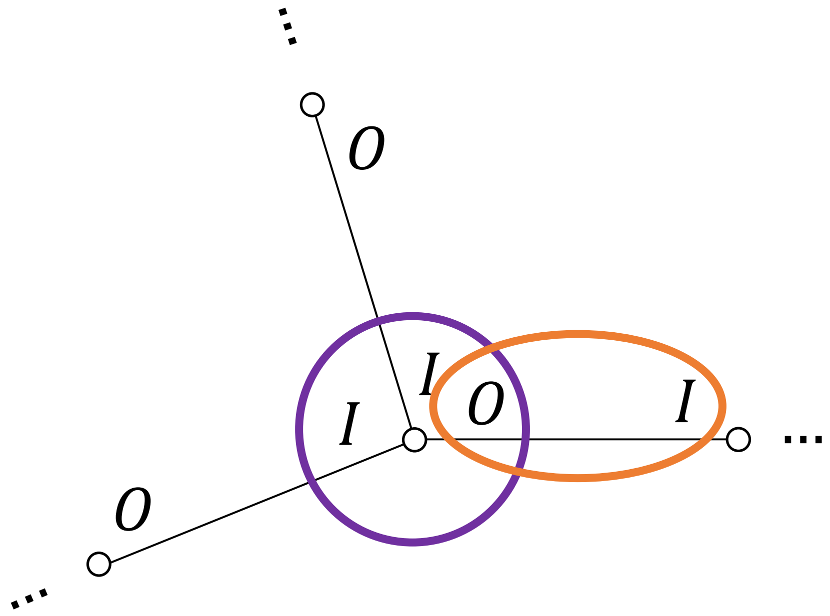
Sinkless Orientation



Under the Hood



Under the Hood



Node Configurations:

{ $0\ 0\ 0$,

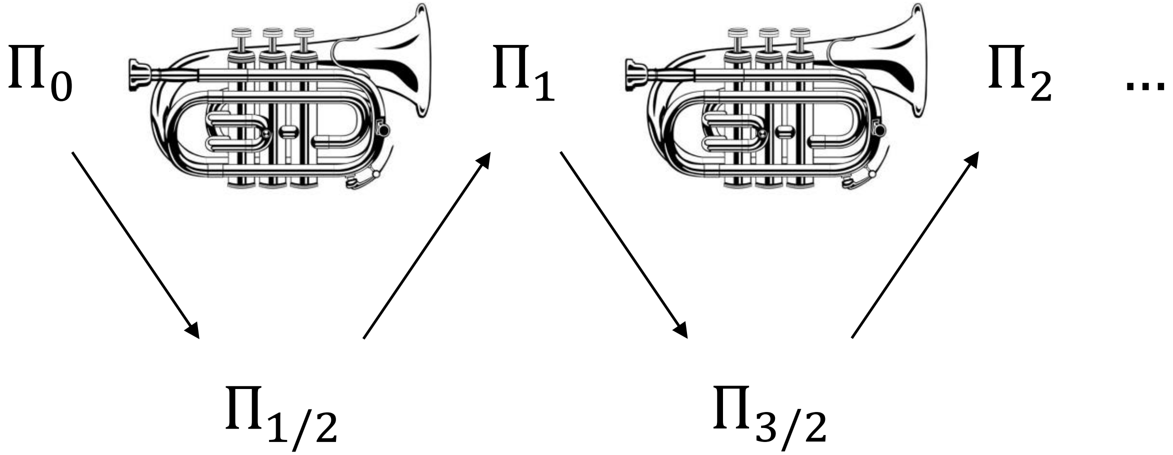
$0\ 0\ I$,

$0\ I\ I$ }

Edge Configurations:

{ $0\ I$ }

Under the Hood



Under the Hood

	Output Labels	Node Configurations	Edge Configurations
Π_0	Y, N	$Y\ \ N\ \ N$	$Y\ \ N,$ $N\ \ N$

Under the Hood

	Output Labels	Node Configurations	Edge Configurations
Π_0	Y, N	$Y\ N\ N$	$Y\ N,$ $N\ N$
	all sets ↓		
$\Pi_{1/2}$	$\{Y\}, \{N\}, \{Y, N\}$		

Under the Hood

	Output Labels	Node Configurations	Edge Configurations
Π_0	Y, N	$Y\ N\ N$	$Y\ N$, $N\ N$
	↓ all sets		↓ $\forall$
$\Pi_{1/2}$	$\{Y\}, \{N\}, \{Y, N\}$		$\{Y, N\}\ \{N\}$, $\{Y\}\ \{N\}$, $\{N\}\ \{N\}$

Under the Hood

	Output Labels	Node Configurations	Edge Configurations
Π_0	Y, N	$Y\ N\ N$	$Y\ N$, $N\ N$
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Under the Hood

	Output Labels	Node Configurations	Edge Configurations
Π_0	Y, N	$Y\ N\ N$	$Y\ N$, $N\ N$
	↓ all sets		↓ $\forall$
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Under the Hood

	Output Labels	Node Configurations	Edge Configurations
Π_0	Y, N	$Y\ N\ N$	$Y\ N,$ $N\ N$
	all sets ↓	$\exists$ ↓	$\forall$ ↓
$\Pi_{1/2}$	$\{Y\}, \{N\}, \{Y, N\}$	$\{Y,N\}\ \{Y,N\}\ \{Y,N\},$ $\{Y,N\}\ \{Y,N\}\ \{N\},$ $\{Y,N\}\ \{N\}\ \{N\},$ $\{Y\}\ \{Y,N\}\ \{Y,N\},$ $\{Y\}\ \{Y,N\}\ \{N\},$ $\{Y\}\ \{N\}\ \{N\}$	$\{Y,N\}\ \{N\},$ $\{Y\}\ \{N\},$ $\{N\}\ \{N\}$

Under the Hood

	Output Labels	Node Configurations	Edge Configurations
Π_0	Y, N	$Y\ N\ N$	$Y\ N$, $N\ N$
	all sets ↓	$\exists$ ↓	$\forall$ ↓
$\Pi_{1/2}$	$\{Y\}, \{N\}, \{Y, N\}$	$\{Y, N\}\ \{Y, N\}\ \{Y, N\}$, $\{Y, N\}\ \{Y, N\}\ \{N\}$, $\{Y, N\}\ \{N\}\ \{N\}$, $\{Y\}\ \{Y, N\}\ \{Y, N\}$, $\{Y\}\ \{Y, N\}\ \{N\}$, $\{Y\}\ \{N\}\ \{N\}$	$\{Y, N\}\ \{N\}$, $\{Y\}\ \{N\}$, $\{N\}\ \{N\}$

Under the Hood

	Output Labels	Node Configurations	Edge Configurations
Π_0	Y, N	$Y\ N\ N$	$Y\ N$, $N\ N$
	all sets ↓	$\exists$ ↓	$\forall$ ↓
$\Pi_{1/2}$	$\{Y\}, \{N\}, \{Y, N\}$	$\{Y,N\}\ \{Y,N\}\ \{Y,N\}$, $\{Y,N\}\ \{Y,N\}\ \{N\}$, $\{Y,N\}\ \{N\}\ \{N\}$, $\{Y\}\ \{Y,N\}\ \{Y,N\}$, $\{Y\}\ \{Y,N\}\ \{N\}$, $\{Y\}\ \{N\}\ \{N\}$	$\{Y,N\}\ \{N\}$, ~~$\{Y\}\ \{N\}$,~~ ~~$\{N\}\ \{N\}$~~

Maximality

Under the Hood

	Output Labels	Node Configurations	Edge Configurations
Π_0	Y, N	$Y\ N\ N$	$Y\ N$, $N\ N$
	all sets ↓	$\exists$ ↓	$\forall$ ↓
$\Pi_{1/2}$	~~$\{Y\}$~~, $\{N\}, \{Y, N\}$	$\{Y,N\}\ \{Y,N\}\ \{Y,N\}$, $\{Y,N\}\ \{Y,N\}\ \{N\}$, $\{Y,N\}\ \{N\}\ \{N\}$, ~~$\{Y\}\ \{Y,N\}\ \{Y,N\}$,~~ ~~$\{Y\}\ \{Y,N\}\ \{N\}$,~~ ~~$\{Y\}\ \{N\}\ \{N\}$~~	$\{Y,N\}\ \{N\}$, ~~$\{Y\}\ \{N\}$,~~ ~~$\{N\}\ \{N\}$~~

Under the Hood

	Output Labels	Node Configurations	Edge Configurations
Π_0	Y, N	$Y\ N\ N$	$Y\ N,$ $N\ N$
	↓ all sets	↓ $\exists$	↓ $\forall$
$\Pi_{1/2}$	$\{N\}, \{Y, N\}$	$\{Y,N\}\ \{Y,N\}\ \{Y,N\},$ $\{Y,N\}\ \{Y,N\}\ \{N\},$ $\{Y,N\}\ \{N\}\ \{N\}$	$\{Y,N\}\ \{N\}$

Under the Hood

	Output Labels	Node Configurations	Edge Configurations
Π_0	Y, N	$Y\ N\ N$	$Y\ N,$ $N\ N$
	all sets ↓	$\exists$ ↓	$\forall$ ↓
$\Pi_{1/2}$	I, O	$O\ O\ O,$ $O\ O\ I,$ $O\ I\ I$	$O\ I$

Under the Hood

	Output Labels	Node Configurations	Edge Configurations
Π_0	Y, N	$Y\ N\ N$	$Y\ N,$ $N\ N$
	all sets ↓	$\exists$ ↓	$\forall$ ↓
$\Pi_{1/2}$	I, O	$O\ O\ O,$ $O\ O\ I,$ $O\ I\ I$	$O\ I$

Sinkless Orientation

Under the Hood

	Output Labels	Node Configurations		Edge Configurations
Π_0	Y, N	$Y\ N\ N$		$Y\ N$, $N\ N$
	all sets $\downarrow$	$\exists \downarrow$		$\forall \downarrow$
$\Pi_{1/2}$	I, O	$O\ O\ O$, $O\ O\ I$, $O\ I\ I$	Sinkless Orientation	$O\ I$
	all sets $\downarrow$	$\forall \downarrow$		$\exists \downarrow$
Π_1	Y, N	$Y\ N\ N$		$Y\ N$, $N\ N$

Maximal Matching

There is no randomized MM algorithm with complexity

$$o(\Delta) + o\left(\frac{\log \log n}{\log \log \log n}\right).$$

There is no deterministic MM algorithm with complexity

$$o(\Delta) + o\left(\frac{\log n}{\log \log n}\right).$$

Maximal Matching

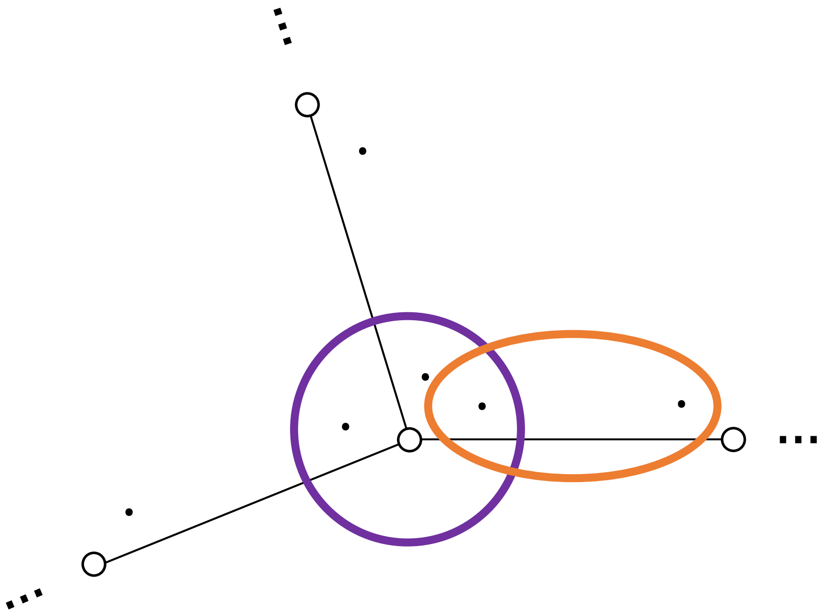
There is no randomized MM algorithm with complexity

$$o(\Delta) + o\left(\frac{\log\log n}{\log\log\log n}\right).$$

There is no deterministic MM algorithm with complexity

$$o(\Delta) + o\left(\frac{\log n}{\log\log n}\right).$$

Bipartite Round Elimination



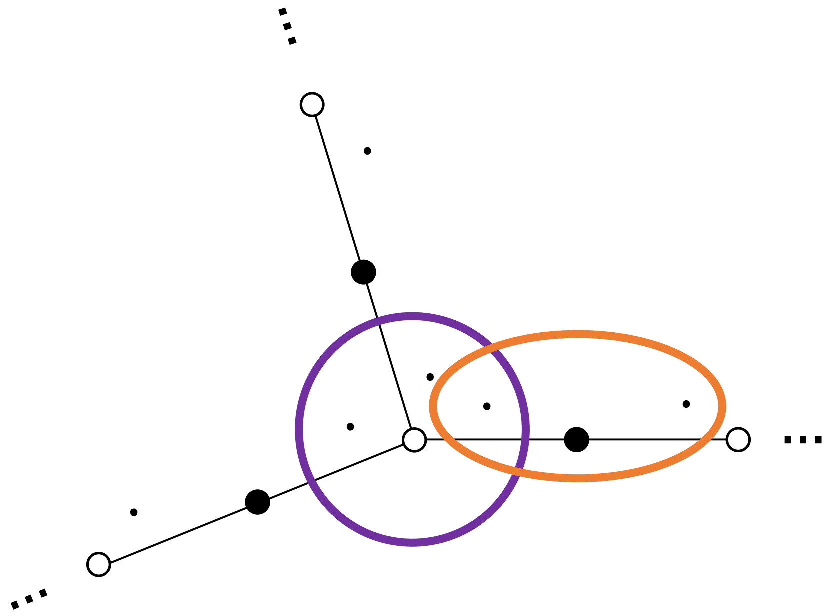
Node Configurations:

{ … }

Edge Configurations:

{ … }

Bipartite Round Elimination



White Configurations:

{ … }

Black Configurations:

{ … }

Maximal Matching

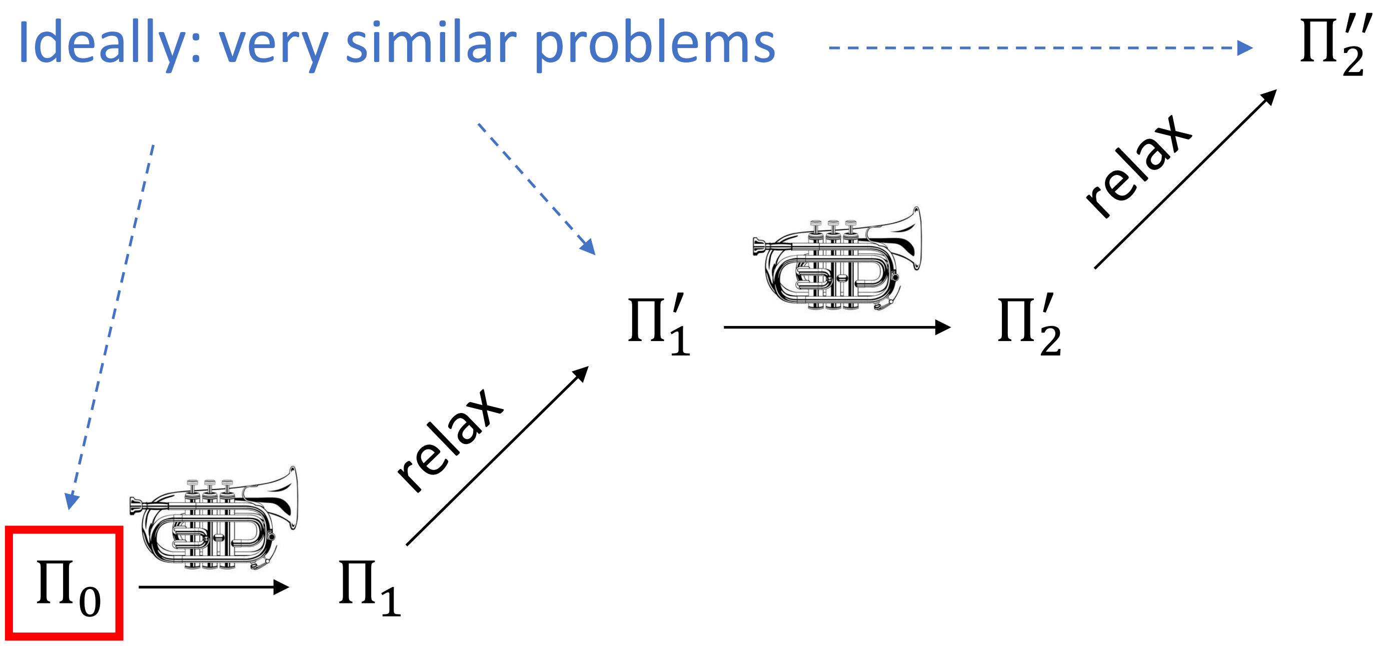
White Configurations:

$$\{ \quad M, O, \dots, O \quad ,$$
$$P, P, \dots, P \quad \}$$

Black Configurations:

$$\{ \quad M, \overset{P}{O}, \dots, \overset{P}{O} \quad ,$$
$$O, O, \dots, O \quad \}$$

Maximal Matching



White Configurations:

$\{ \quad M, O, \dots, O \quad ,$

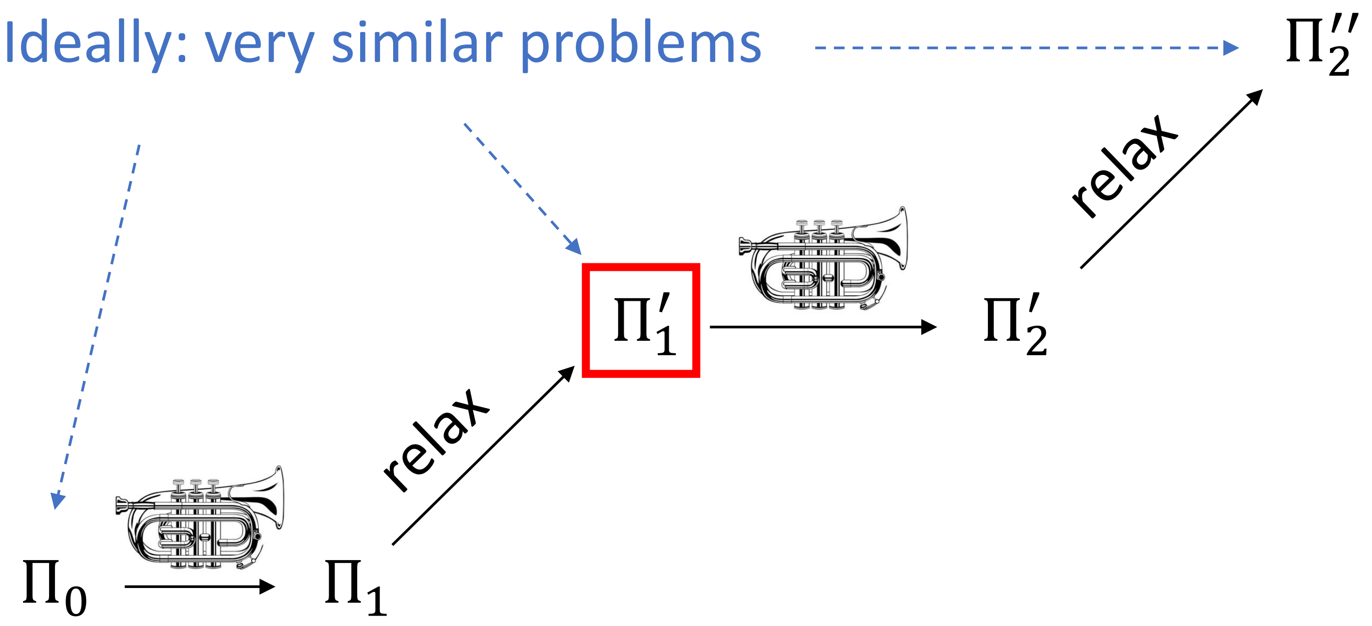
$\quad P, P, \dots, P \quad \}$

Black Configurations:

$\{ \quad M, \substack{P \\ O}, \dots, \substack{P \\ O} \quad ,$

$\quad O, O, \dots, O \quad \}$

Maximal Matching



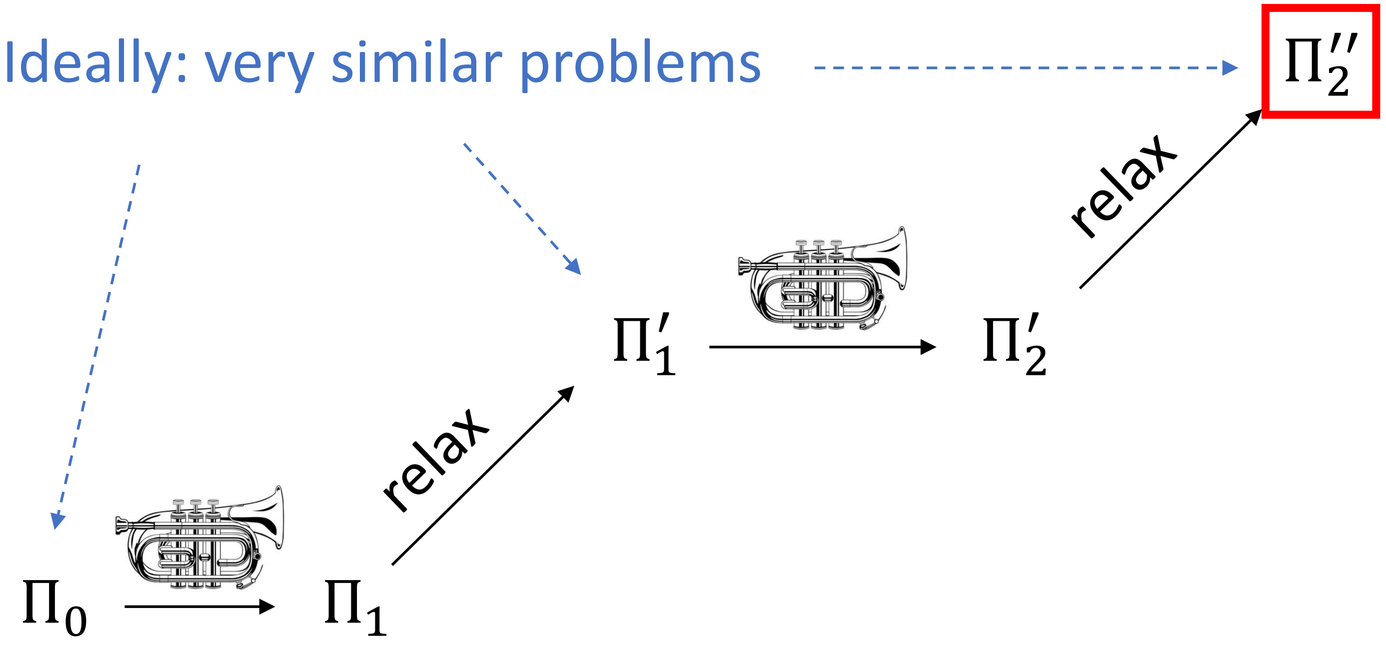
White Configurations:

$$\{ \quad M, O, \ldots, O, X \quad ,$$
$$P, P, \ldots, P, X \quad \}$$

Black Configurations:

$$\{ \quad M, \genfrac{}{}{0pt}{}{P}{O}, \ldots, \genfrac{}{}{0pt}{}{P}{O}, X \quad ,$$
$$O, O, \ldots, O, X \quad ,$$
$$\ldots \quad \}$$

Maximal Matching



White Configurations:

$\{\quad M, O, \ldots, O, X, X, Y \quad ,$

$\quad P, P, \ldots, P, X, X, Y \quad \}$

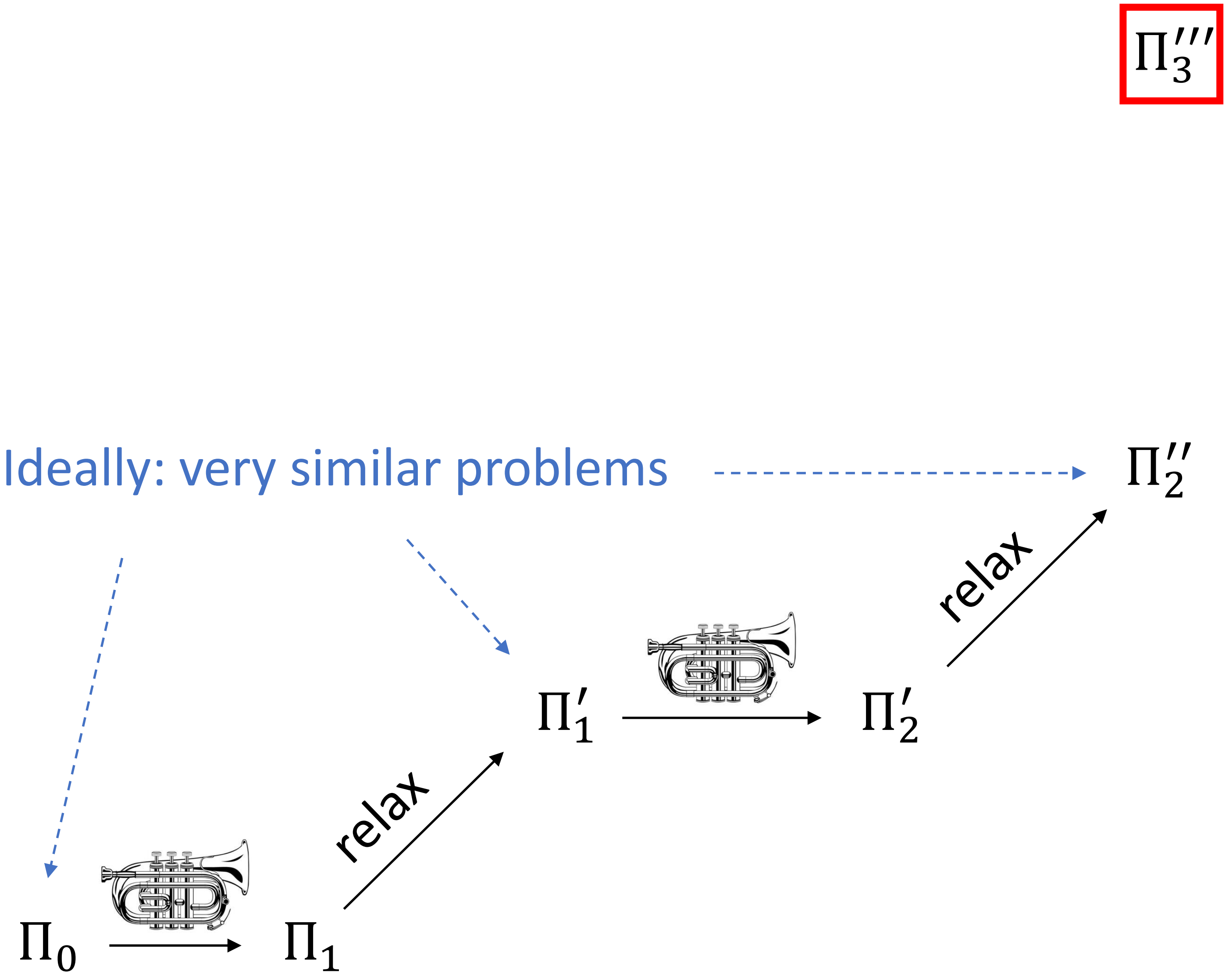
Black Configurations:

$\{\quad M, \genfrac{}{}{0pt}{}{P}{O}, \ldots, \genfrac{}{}{0pt}{}{P}{O}, X, X, Y \quad ,$

$\quad O, O, \ldots, O, X, X, Y \quad ,$

$\quad \ldots \quad \}$

Maximal Matching



White Configurations:

$\{ \quad M, O, \ldots, O, X, X, Y, X, Y, Y,$

$P, P, \ldots, P, X, X, Y, X, Y, Y \quad \}$

Black Configurations:

$\{ \quad M, \substack{P \\ O}, \ldots, \substack{P \\ O}, X, X, Y, X, Y, Y \ ,$

$O, O, \ldots, O, X, X, Y, X, Y, Y \ ,$

$\ldots \quad \}$

Maximal Matching

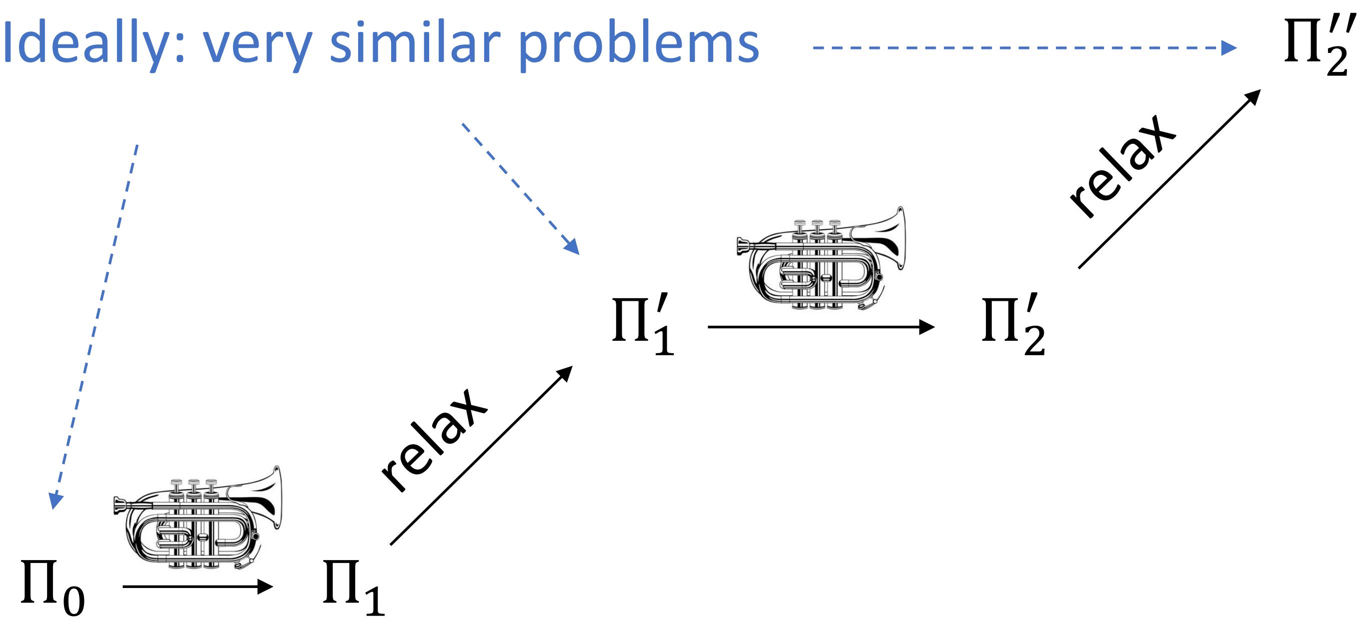
Π_3'''

In each step:

$$\#X_{\text{new}} = \#X_{\text{old}} + 1$$

$$\#Y_{\text{new}} = \#Y_{\text{old}} + \#X_{\text{old}}$$

Ideally: very similar problems



White Configurations:

$\{ \quad M, O, \ldots, O, X, X, Y, X, Y, Y,$

$P, P, \ldots, P, X, X, Y, X, Y, Y \quad \}$

Black Configurations:

$\{ \quad M, {}^{P}_{O}, \ldots, {}^{P}_{O}, X, X, Y, X, Y, Y,$

$O, O, \ldots, O, X, X, Y, X, Y, Y,$

$\ldots \quad \}$

Maximal Matching

In each step:

$$\#X_{\text{new}} = \#X_{\text{old}} + 1$$

$$\#Y_{\text{new}} = \#Y_{\text{old}} + \#X_{\text{old}}$$

White Configurations:

$\{ \quad M, O, \dots, O \quad ,$

$P, P, \dots, P \quad \}$

Black Configurations:

$\{ \quad M, {}^{P}_{O}, \dots, {}^{P}_{O} \quad ,$

$O, O, \dots, O \quad \}$

If $\#X + \#Y < \frac{\Delta}{2}$, then there exists no 0-round algorithm for the problem.

→ No $o(\sqrt{\Delta})$-round algorithm for Π_0 = Maximal Matching.

Maximal Matching

In each step:

$$\#X_{\text{new}} = \#X_{\text{old}} + 1$$

$$\#Y_{\text{new}} = \#Y_{\text{old}} + \#X_{\text{old}}$$

New Π_0

White Configurations:

$\{ \quad M, O, \dots, O, X, \dots, X,$

$P, P, \dots, P, \underbrace{X, \dots, X}_{\sqrt{\Delta}} \quad \}$

Black Configurations:

$\{ \quad M, {}^{P}_{O}, \dots, {}^{P}_{O}, X, \dots, X,$

$O, O, \dots, O, X, \dots, X \quad \}$

If $\#X + \#Y < \frac{\Delta}{2}$, then there exists no 0-round algorithm for the problem.

No $o(\sqrt{\Delta})$-round algorithm for Π_0 = Maximal Matching.

Maximal Matching

In each step:

$$\#X_{\text{new}} = \#X_{\text{old}} + 1$$

$$\#Y_{\text{new}} = \#Y_{\text{old}} + \#X_{\text{old}}$$

New Π_0

White Configurations:

$\{\ M, O, \dots, O, X, \dots, X,$

$P, P, \dots, P, \underbrace{X, \dots, X}_{\sqrt{\Delta}}\ \}$

Black Configurations:

$\{\ M, {}^{P}_{O}, \dots, {}^{P}_{O}, X, \dots, X,$

$O, O, \dots, O, X, \dots, X\ \}$

If $\#X + \#Y < \frac{\Delta}{2}$, then there exists no 0-round algorithm for the problem.

No $o(\sqrt{\Delta})$-round algorithm for $\Pi_0 =$ Maximal $\sqrt{\Delta}$ -Matching.

Maximal Matching

Maximal $\sqrt{\Delta}$ -Matching = Maximal Matching, where each node can be matched to up to $\sqrt{\Delta}$ neighbors.

Maximal Matching

Maximal $\sqrt{\Delta}$ -Matching = Maximal Matching, where each node can be matched to up to $\sqrt{\Delta}$ neighbors.

$o(\Delta)$-round algorithm for maximal matching

$\Downarrow$

$o(\sqrt{\Delta})$-round algorithm for maximal $\sqrt{\Delta}$ -matching

Maximal Matching

Maximal $\sqrt{\Delta}$ -Matching = Maximal Matching, where each node can be matched to up to $\sqrt{\Delta}$ neighbors.

no $o(\Delta)$-round algorithm for maximal matching

$\Uparrow$

no $o(\sqrt{\Delta})$-round algorithm for maximal $\sqrt{\Delta}$ -matching

Maximal Matching

no $o(\Delta)$-round algorithm for maximal matching

$\Downarrow$ ← explicitly incorporating error probabilities

no randomized $o(\Delta) + o\left(\frac{\log\log n}{\log\log\log n}\right)$-round algorithm for maximal matching

$\Downarrow$ ← different techniques

no deterministic $o(\Delta) + o\left(\frac{\log n}{\log\log n}\right)$-round algorithm for maximal matching

Current Limitations

lower bound technique

for deterministic algorithms

in the LOCAL model

without unique IDs

for locally checkable problems

on high-girth graphs

Current Limitations

lower bound technique

for deterministic algorithms

in the LOCAL model

without unique IDs

for locally checkable problems

crucial ingredient!

on high-girth graphs

Current Limitations

lower bound technique

for deterministic algorithms

in the LOCAL model

without unique IDs

for locally checkable problems

on high-girth graphs

general randomized
round elimination?

Current Limitations

lower bound technique

for deterministic algorithms

in the LOCAL model

without unique IDs

for locally checkable problems

on high-girth graphs

round elimination for other "local" models?

Current Limitations

lower bound technique

for deterministic algorithms

in the LOCAL model

without unique IDs

for locally checkable problems

on high-girth graphs

general guarantees?

Current Limitations

lower bound technique

for deterministic algorithms

in the LOCAL model

without unique IDs

for locally checkable problems

on high-girth graphs

nothing better than $\Omega(\log n)$ possible

$O(\text{poly} \log n)$ known for many problems

[Rozhon, Ghaffari ’19+]

Current Limitations

lower bound technique

not only!

for deterministic algorithms

in the LOCAL model

without unique IDs

for locally checkable problems

on high-girth graphs

Lower Bounds

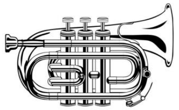
Π_k^*

0

Much simpler description than Π_1

...

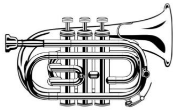
relax



Π'_1 → Π'_2

relax

$T(n, \Delta) - ?$ $\quad$ $T(n, \Delta) - ? - 1$



Problem $\quad \Pi_0 \rightarrow \Pi_1$

Complexity $\quad T(n, \Delta) \quad T(n, \Delta) - 1$

Π_0 has complexity at least k.

Upper Bounds

Π_k^*

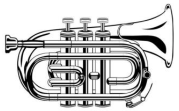
0

Much simpler description than Π_1

...

harden

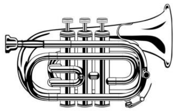
Π_1'



Π_2'

harden

$T(n,\Delta) - 1 + ?$ $T(n,\Delta) - 2 + ?$

Problem	Π_0  $\rightarrow$	Π_1
Complexity	$T(n,\Delta)$	$T(n,\Delta) - 1$

Π_0 has complexity at most k.

Current Limitations

lower bound technique

for deterministic algorithms

in the LOCAL model

without unique IDs

for locally checkable problems

on high-girth graphs

The Future

new bounds for more problems

$(\Delta + 1)$-vertex coloring?

$(2\Delta - 1)$-edge coloring?

The Future

new bounds for more problems

$(\Delta + 1)$-vertex coloring?

$(2\Delta - 1)$-edge coloring?

Odd-Degree Weak 2-Coloring ✓

[B., PODC'19]

The Future

new bounds for more problems

understand the process

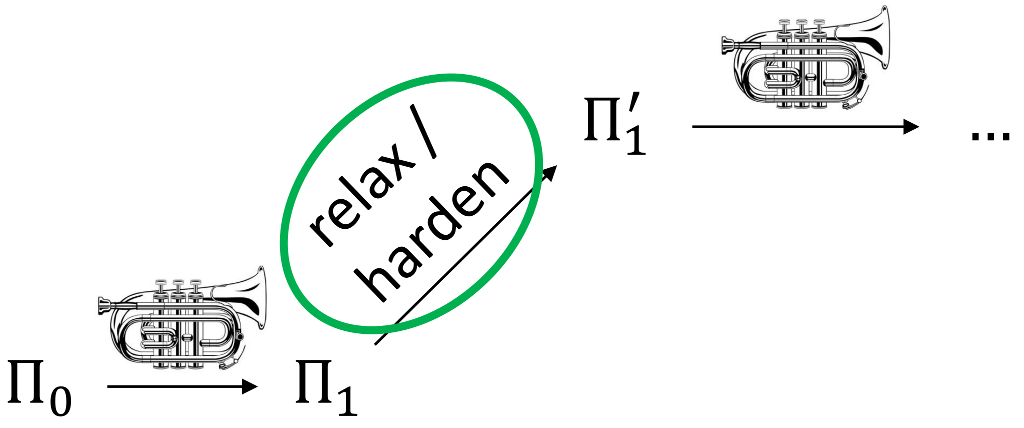
	Node Configurations	Edge Configurations
Π_0	...	...
	$\exists \downarrow$	$\forall \downarrow$
$\Pi_{1/2}$	...	...
	$\forall \downarrow$	$\exists \downarrow$
Π_1	...	...

The Future

new bounds for more problems

understand the process

understand relaxations



The Future

new bounds for more problems

understand the process

understand relaxations

complete classification of
locally checkable problems
on high-girth graphs??

Performed speedup.

Close

https://github.com/olidennis/round-eliminator

[Dennis Olivetti, ’19+]

The problem is NOT zero rounds solvable.

Active (Before Renaming)
Any choice satisfies previous Passive

Passive (Before Renaming)
Exists choice satisfying previous Active

Renaming
Old and new labels

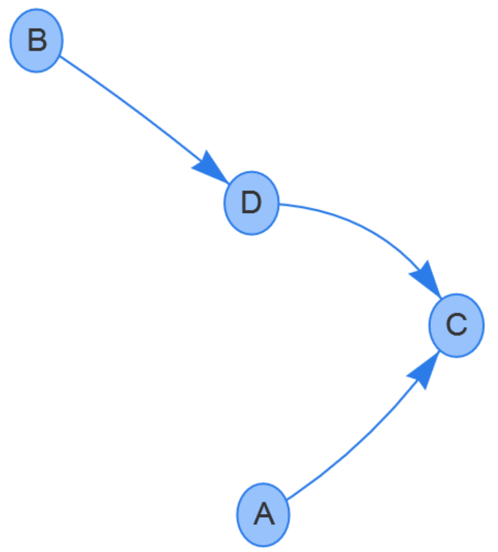
Active
Any choice satisfies previous Passive

D	D	D	A
C	B	B	B

Passive
Exists choice satisfying previous Active

BCD	BCD	BCD	AC
D	D	D	D

Diagram
Strength of right side labels



Tools
Speedup, edit, simplifications, ...

Speedup
Edit
Simplifications
Harden
Automatic Lower Bound
Automatic Upper Bound
New Renaming