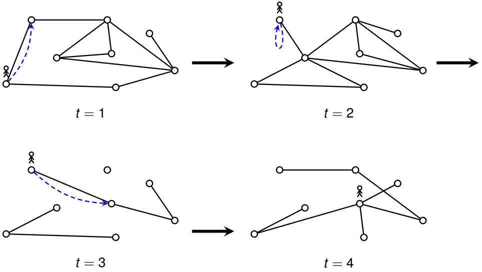


Random walks on dynamic graphs: Mixing times, hitting times, and return probabilities

Thomas Sauerwald and Luca Zanetti

8th Workshop on Advances in Distributed Graph Algorithms 2019

14 Oct 2019

ADGA 2019
8th Workshop on Advances in Distributed Graph Algorithms

UNIVERSITY OF CAMBRIDGE

Outline

Intro

Random Walks on Sequences of Connected Graphs

Random Walks on Sequences of (Possibly) Disconnected Graphs

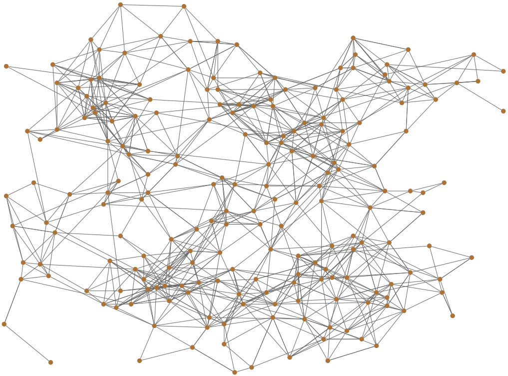
Conclusion

Random Walks and Markov Chains

Random Walks on Graphs

A class of Markov chains where a particle is moving on the vertices of a graph:

- start from some specified vertex
- at each step, jump to a randomly chosen neighbor



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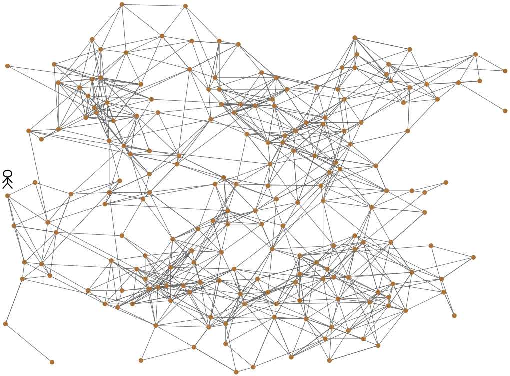

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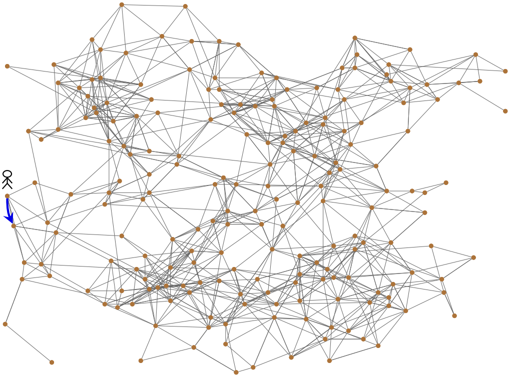

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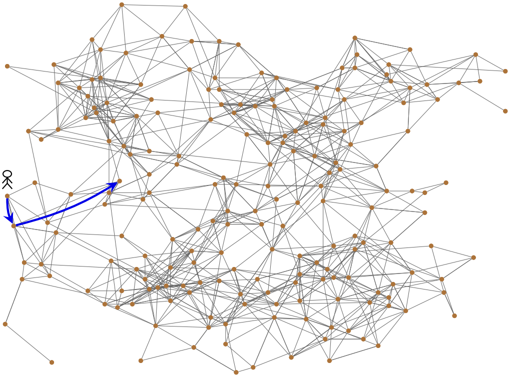

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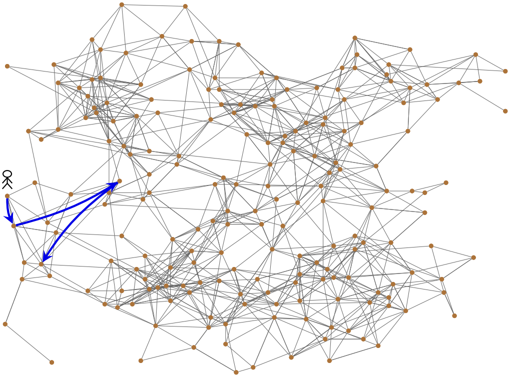

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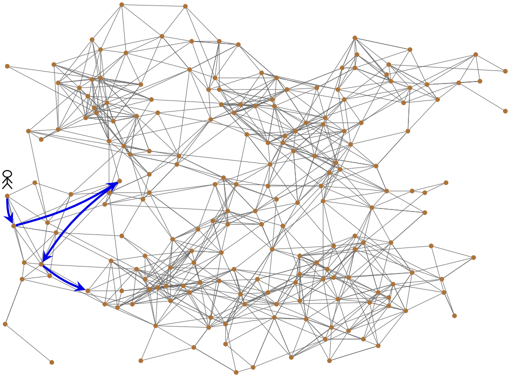

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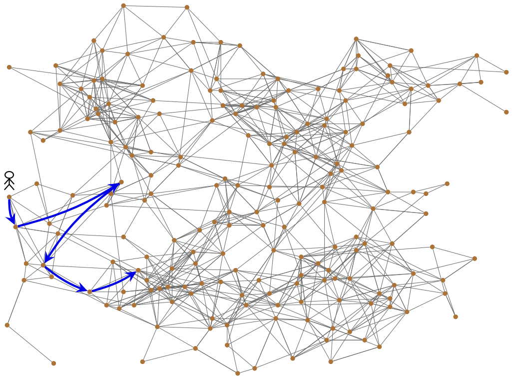

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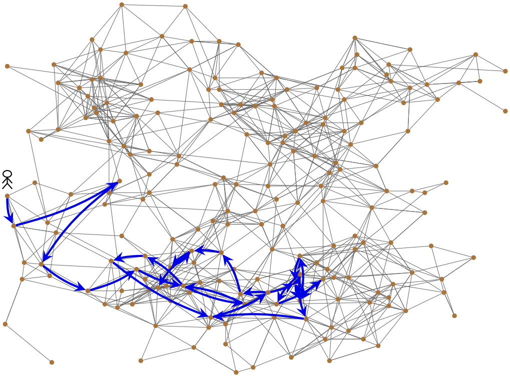

Hitting Times (and Cover Times) on Static Graphs

Hitting and Cover Times

- Let $t_{hit}(u, v)$ be the expected time for a random walk to go from u to v
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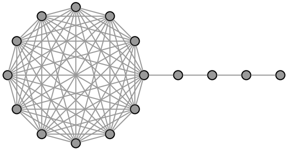

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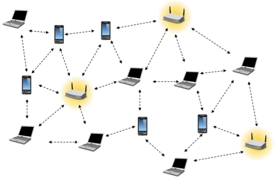

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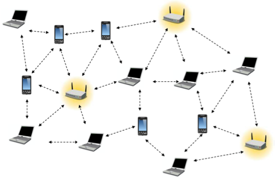

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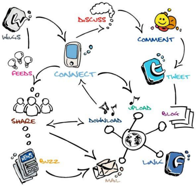

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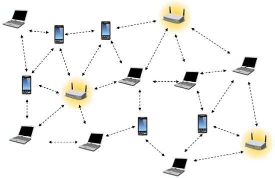

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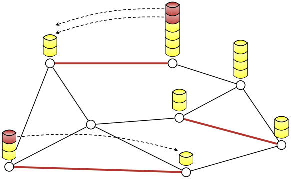
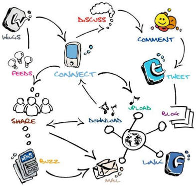


(Distributed) Algorithms



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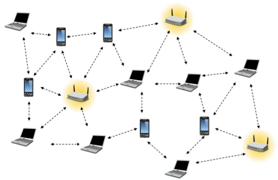

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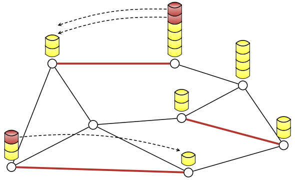

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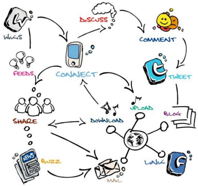

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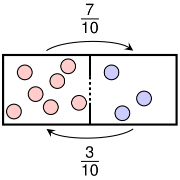

Particle Processes

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Lazy Random Walks

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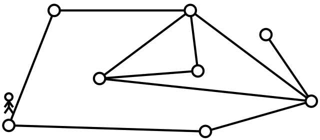

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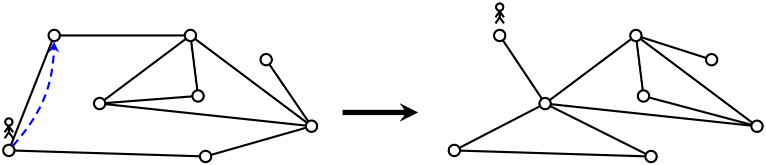

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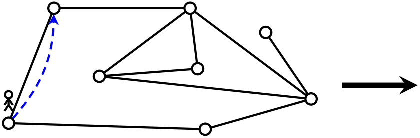

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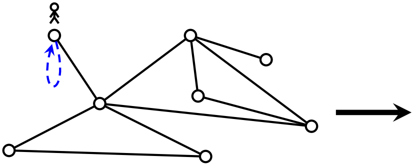

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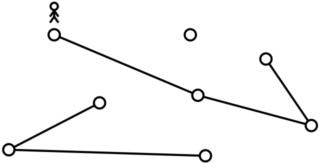

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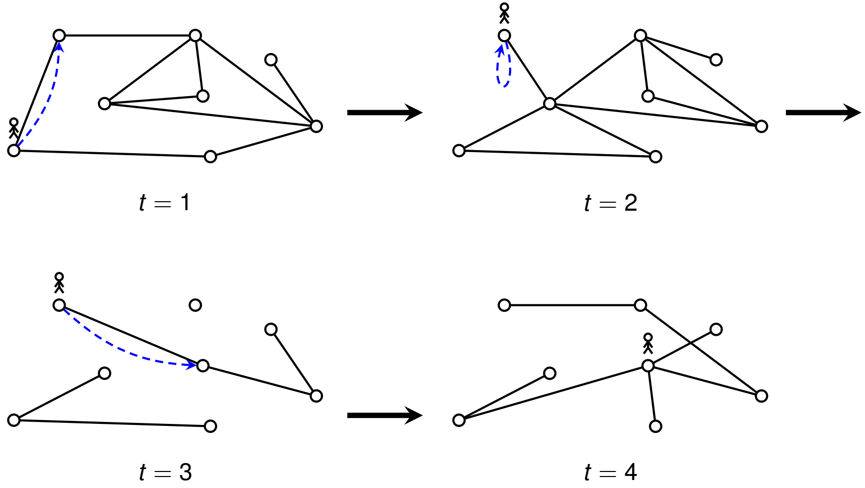

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For static connected graphs:

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- **general case**: $O(n^3)$ mixing and hitting times

For dynamic connected graphs:

- If $\pi^{(t)}$ changes over time, in general, we don't have mixing
- Can we at least say something about hitting times?

Related Work: A Dichotomy for dynamic graphs

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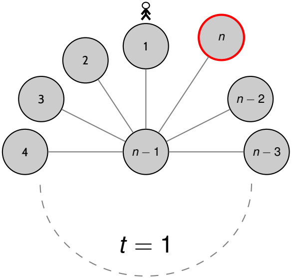

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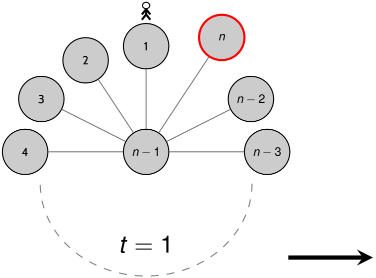

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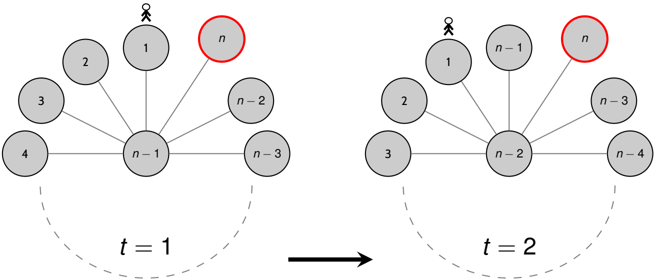

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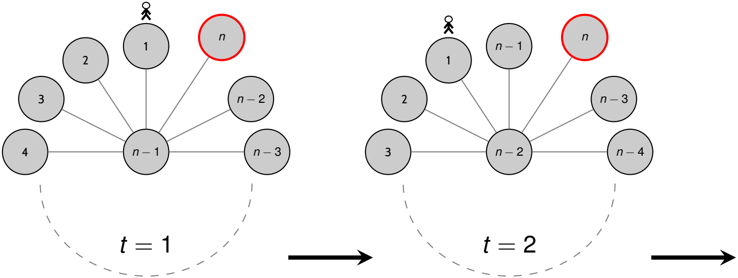

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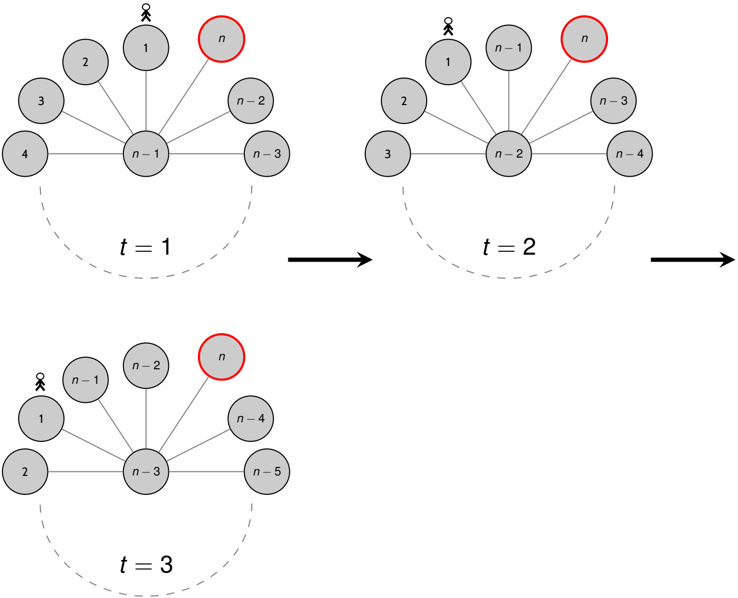

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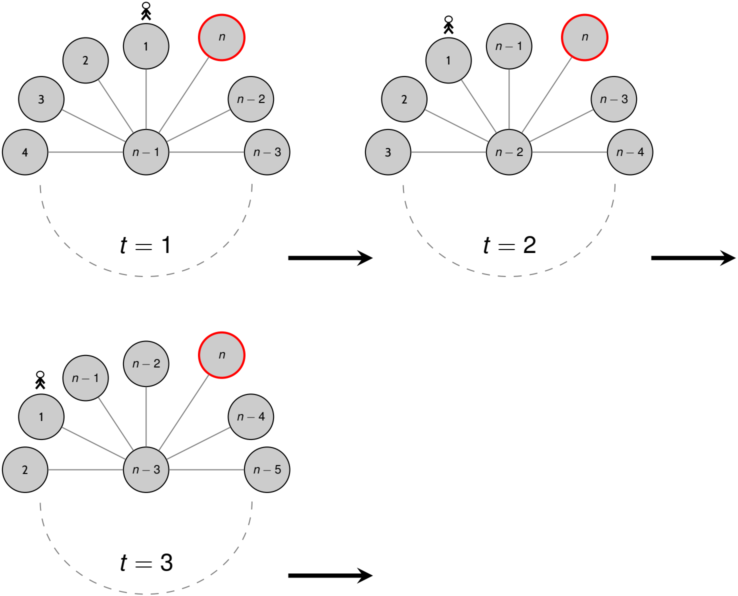

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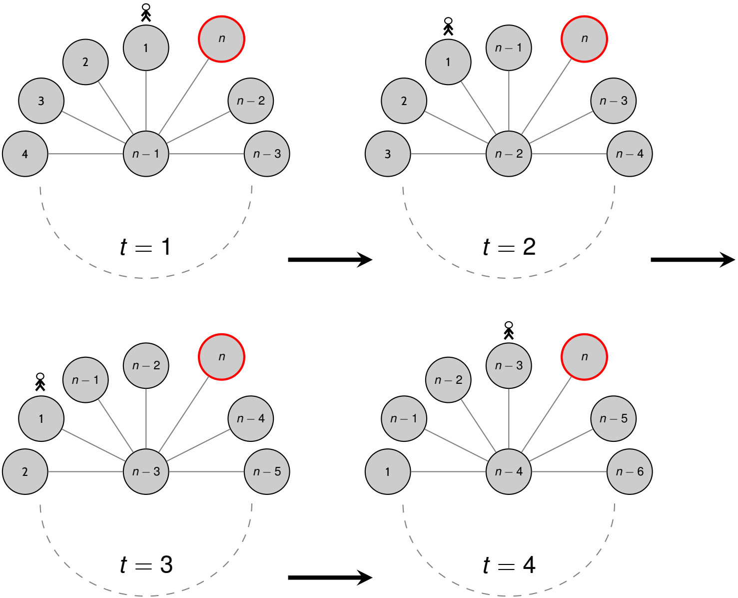

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How can we derive these results?

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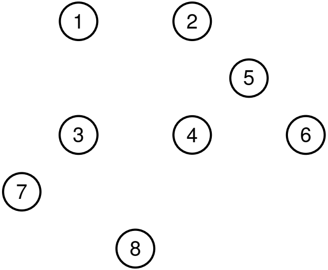
Proof:

- Take a spanning tree T in G



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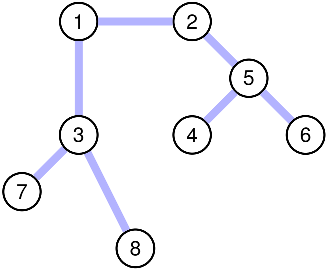

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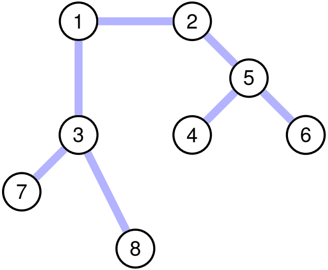

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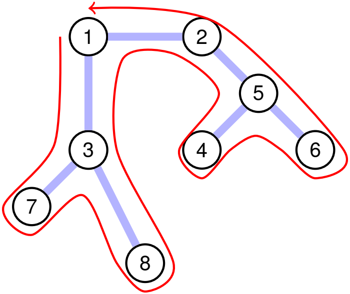

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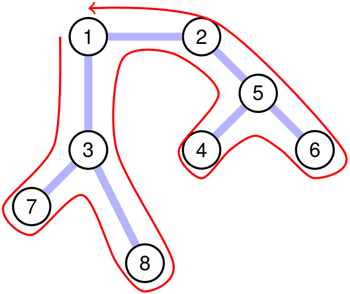

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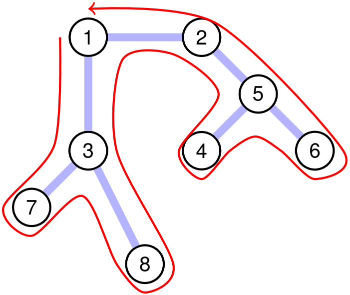

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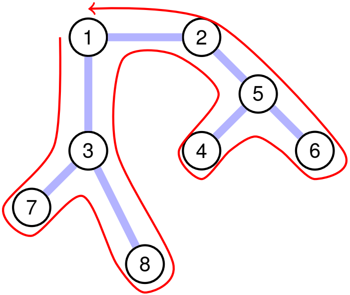

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Both proofs crucially rely on a static spanning tree or static shortest path!

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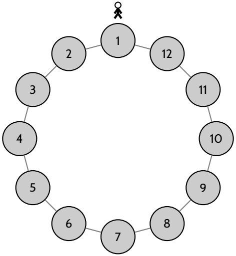

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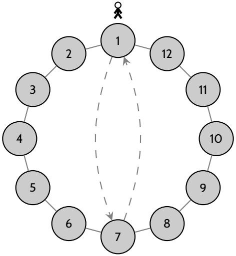

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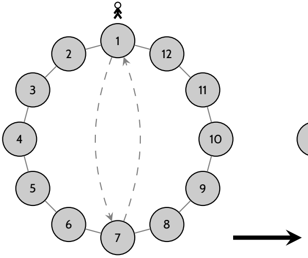

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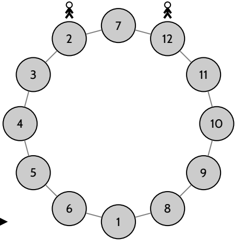

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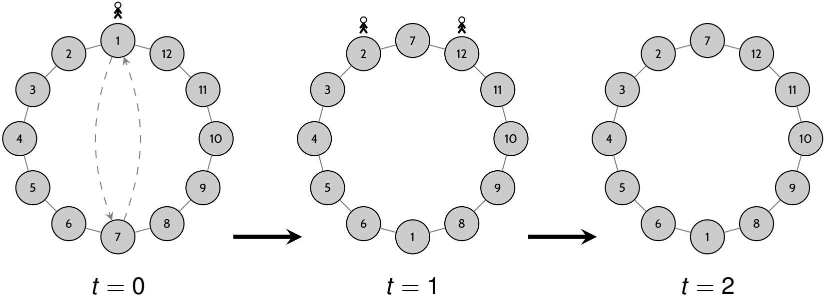

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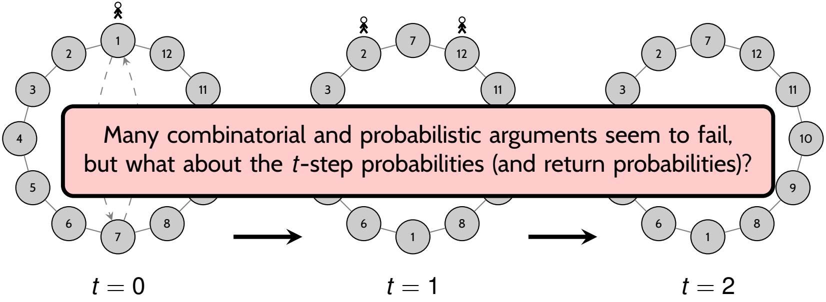
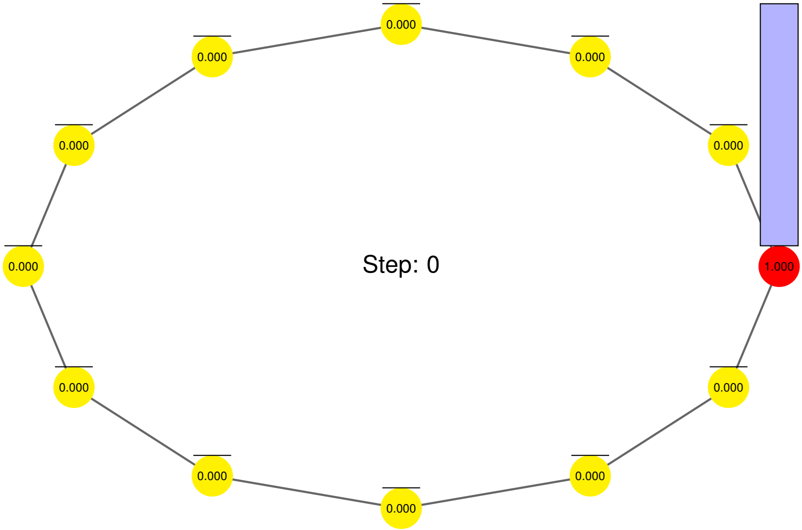


Many combinatorial and probabilistic arguments seem to fail, but what about the t-step probabilities (and return probabilities)?

Diffusion of a Random Walk on a Static Cycle



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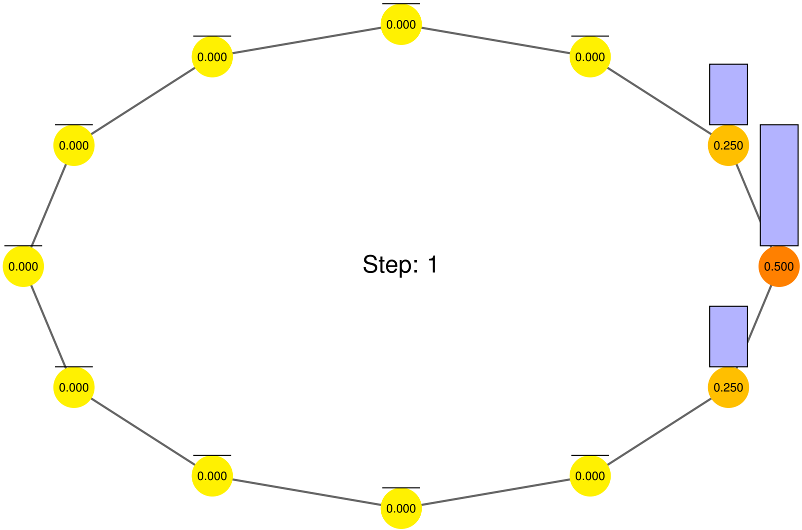

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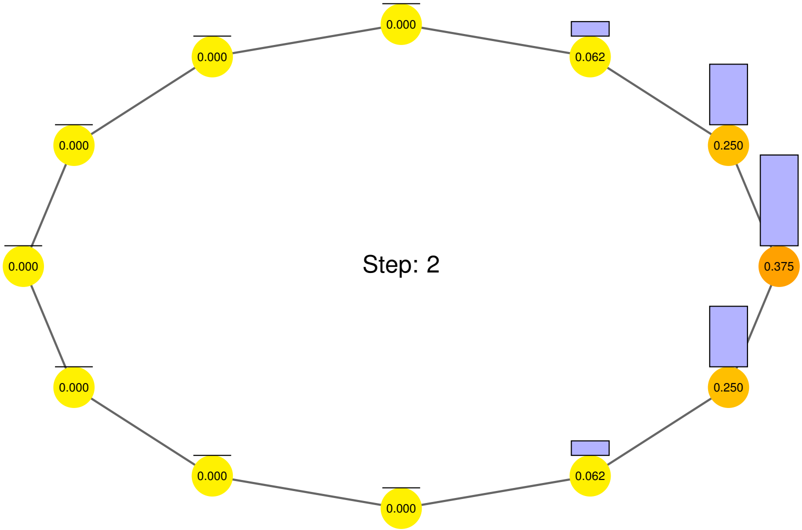

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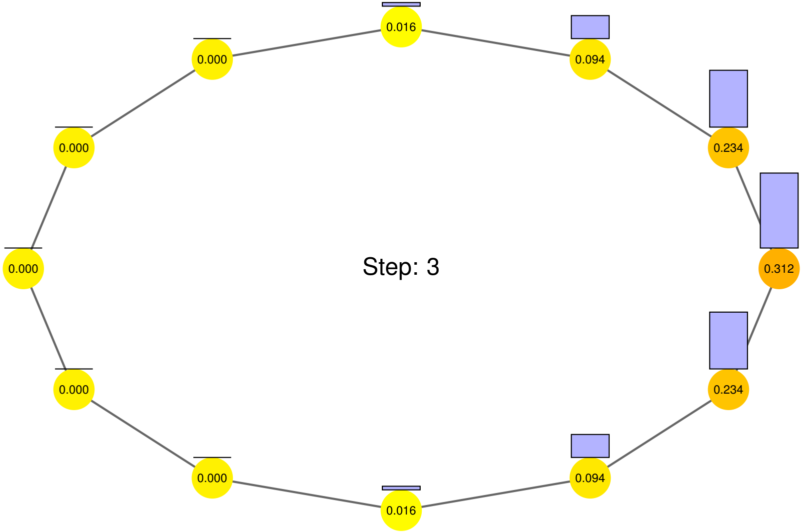

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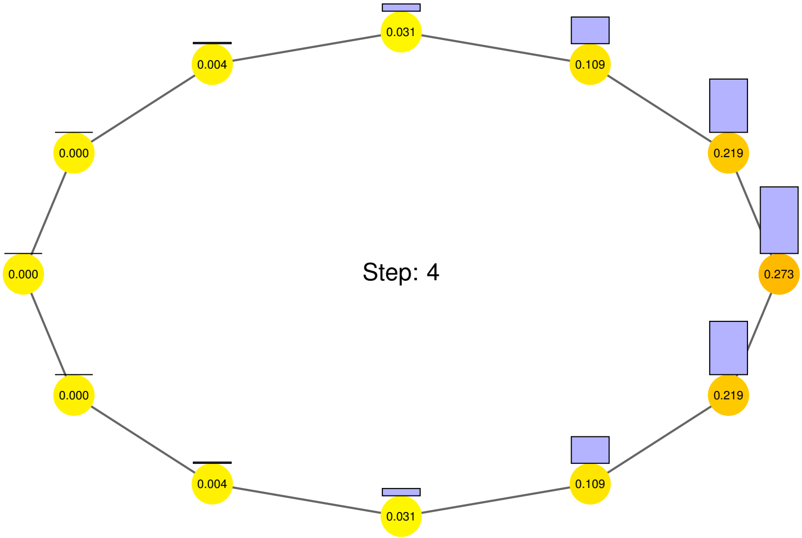

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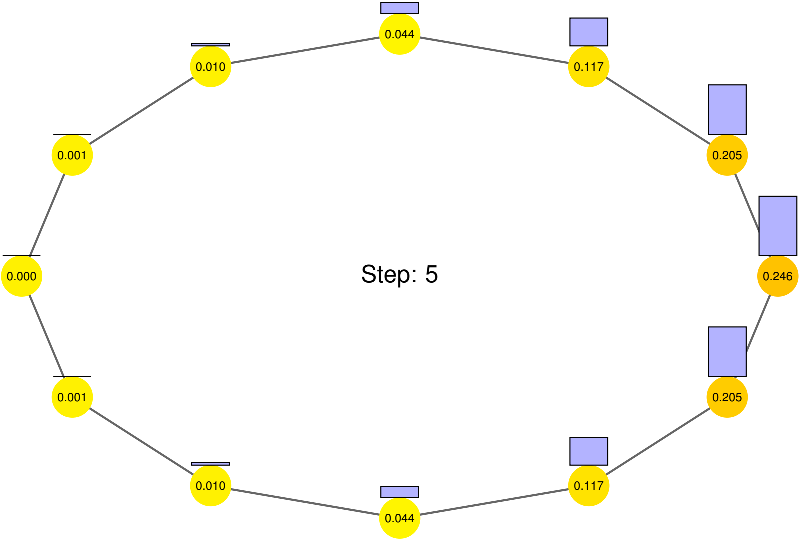

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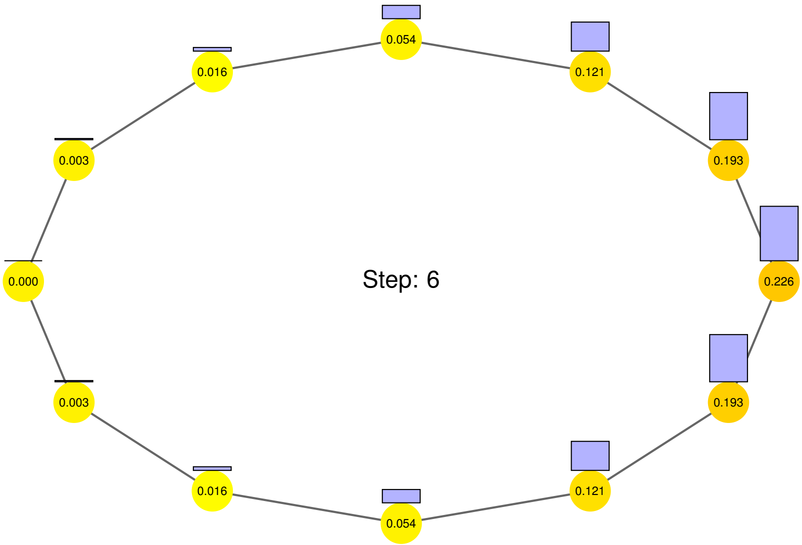

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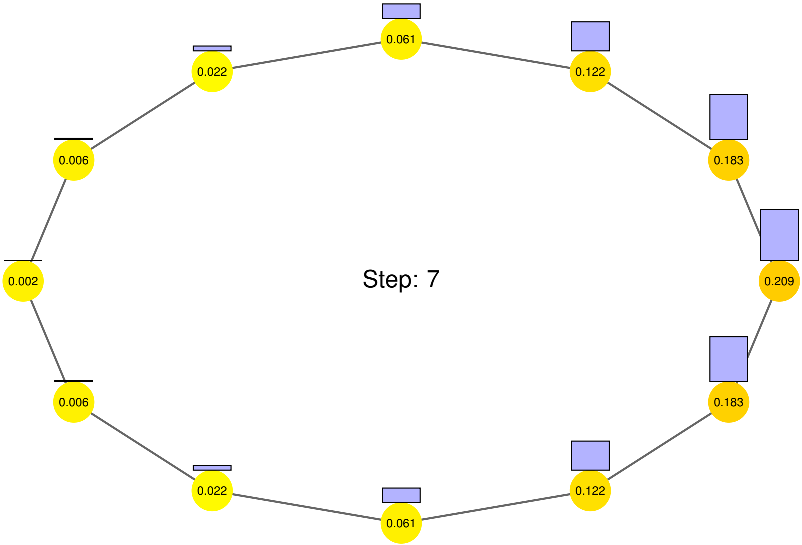

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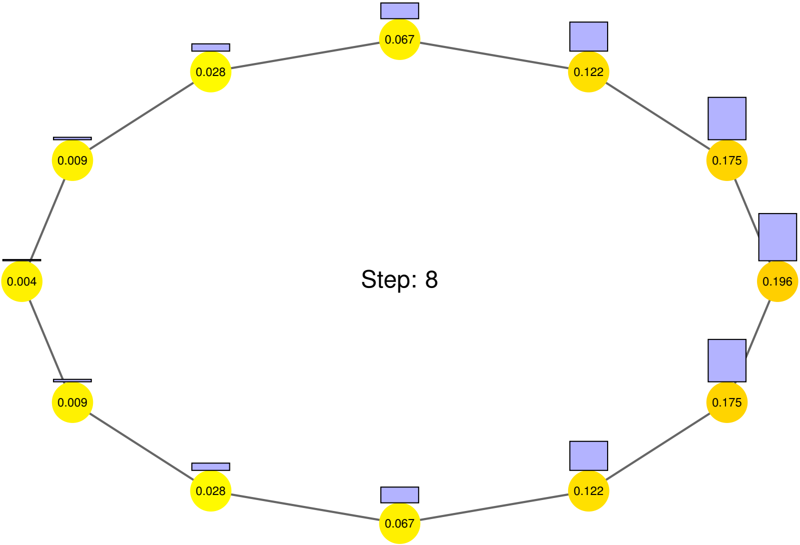

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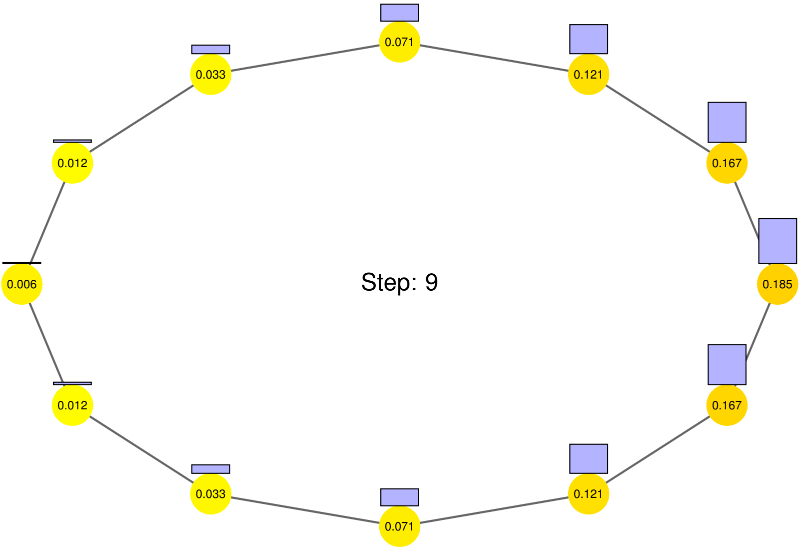

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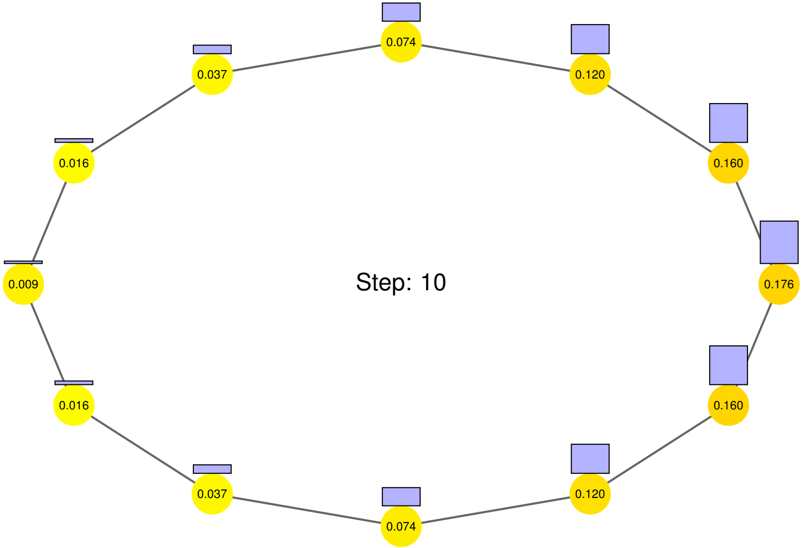

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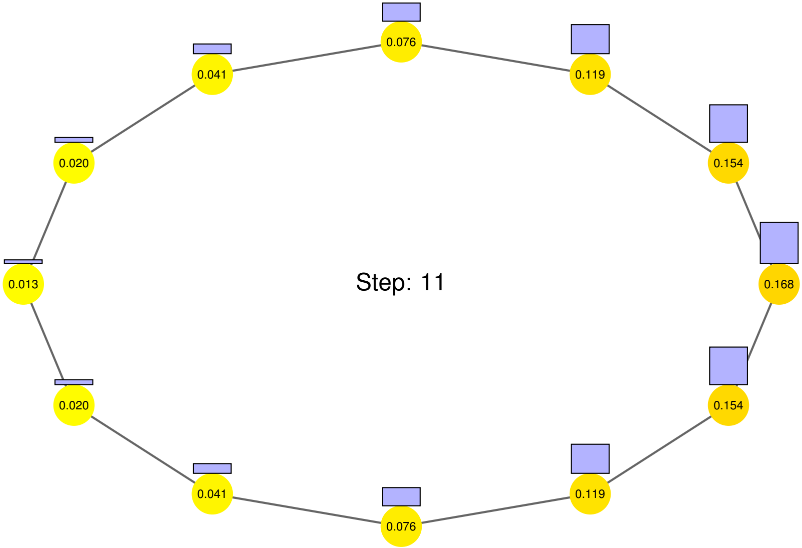

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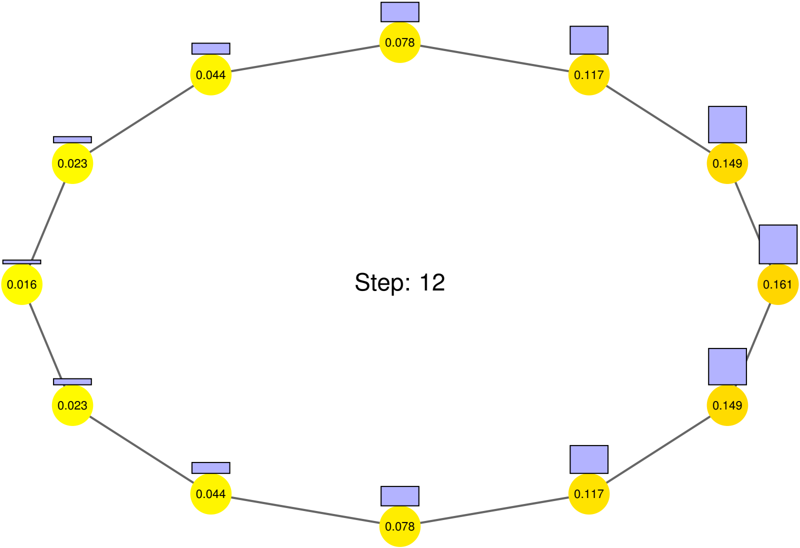

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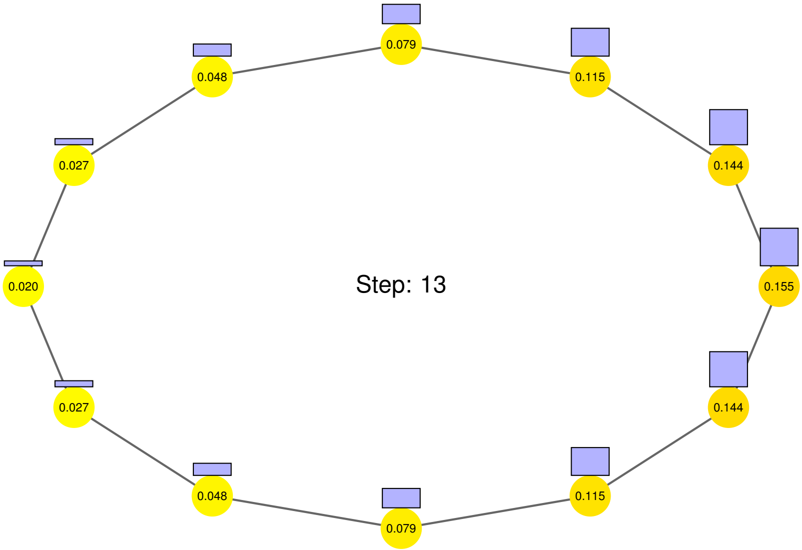

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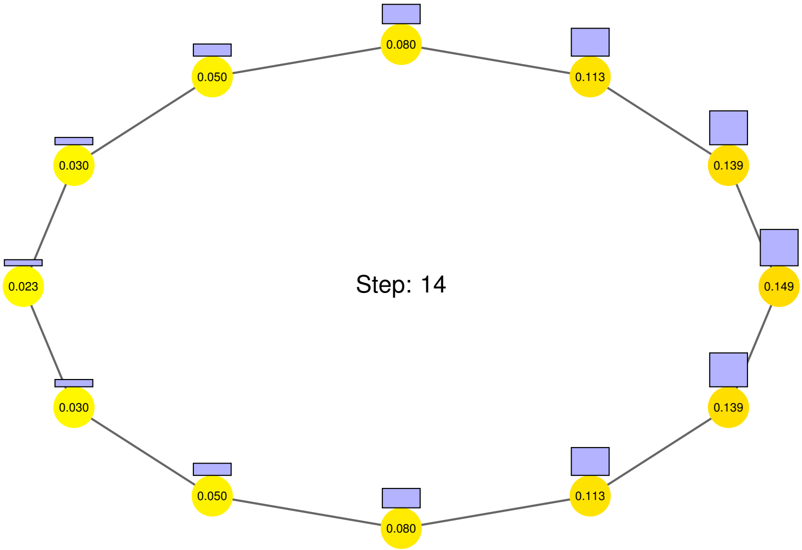

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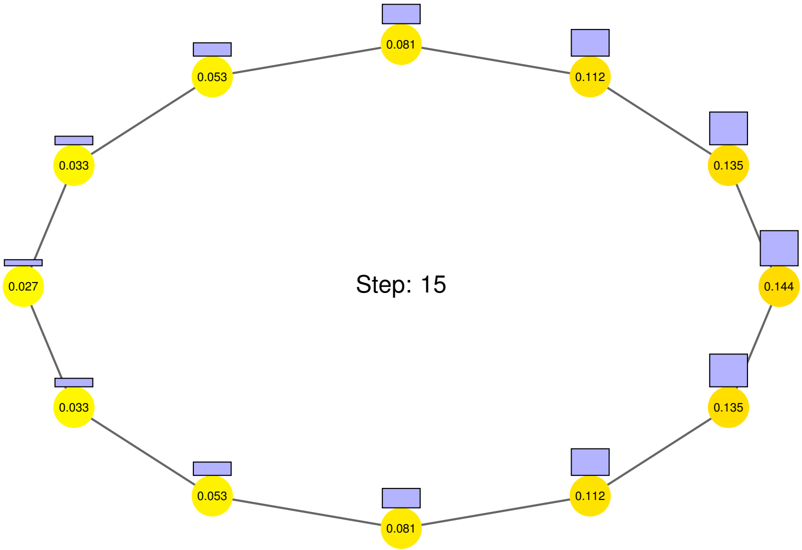

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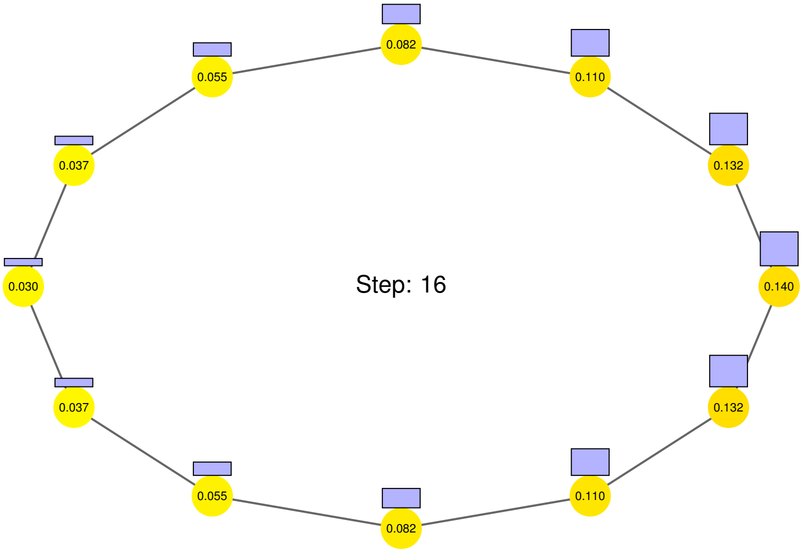

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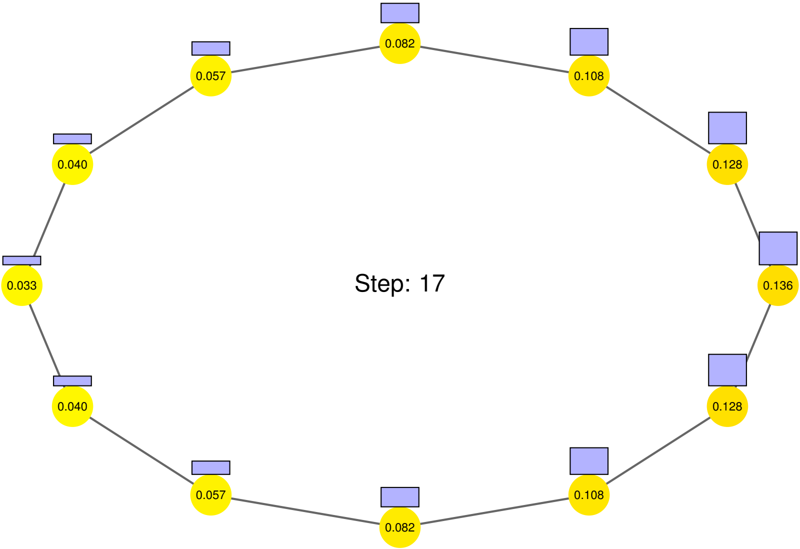

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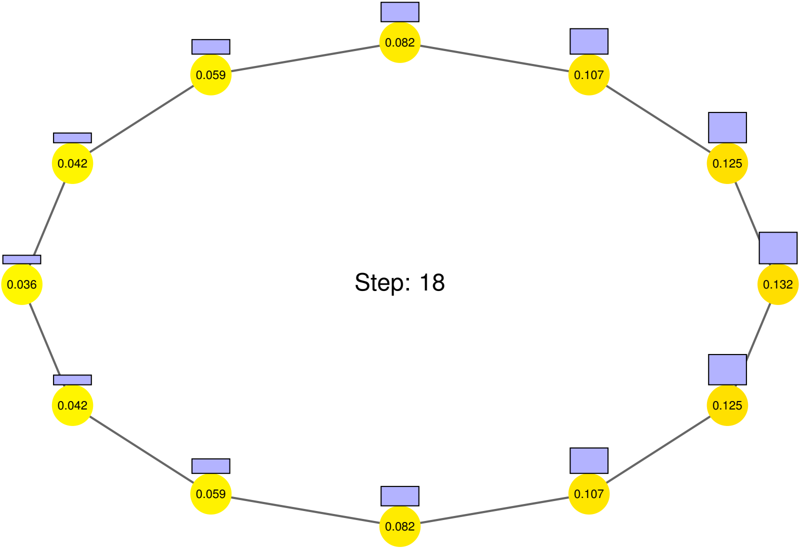

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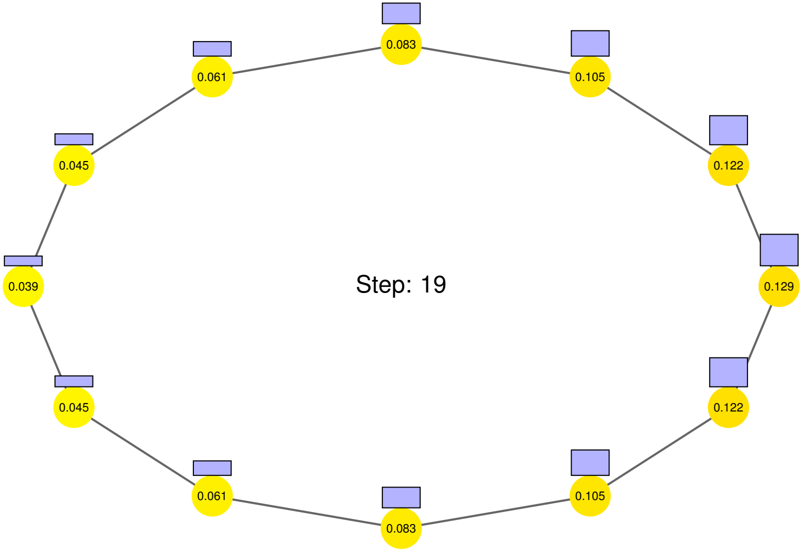

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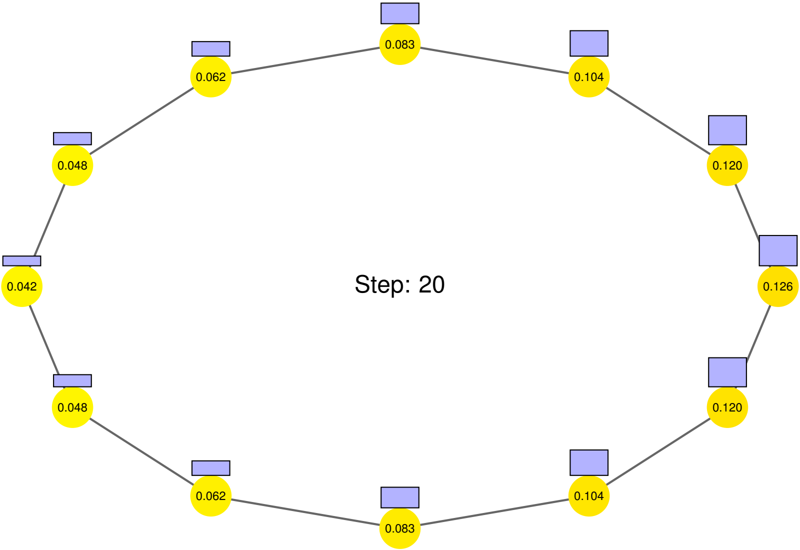

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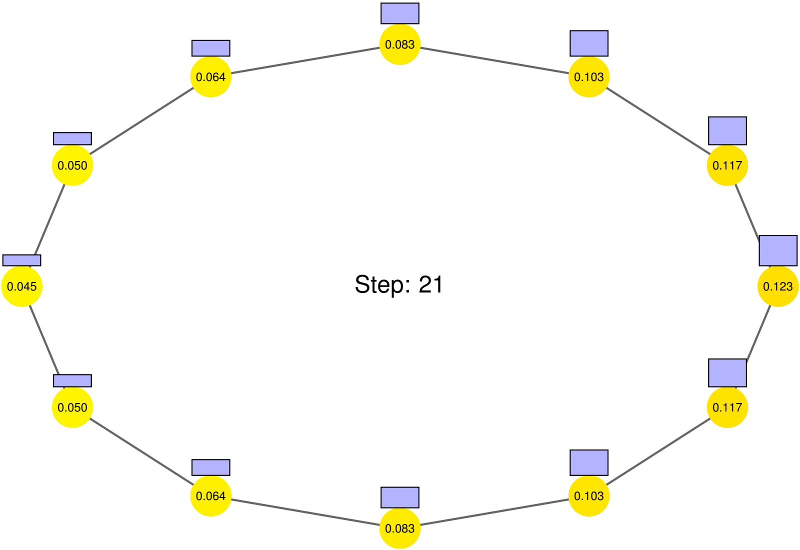

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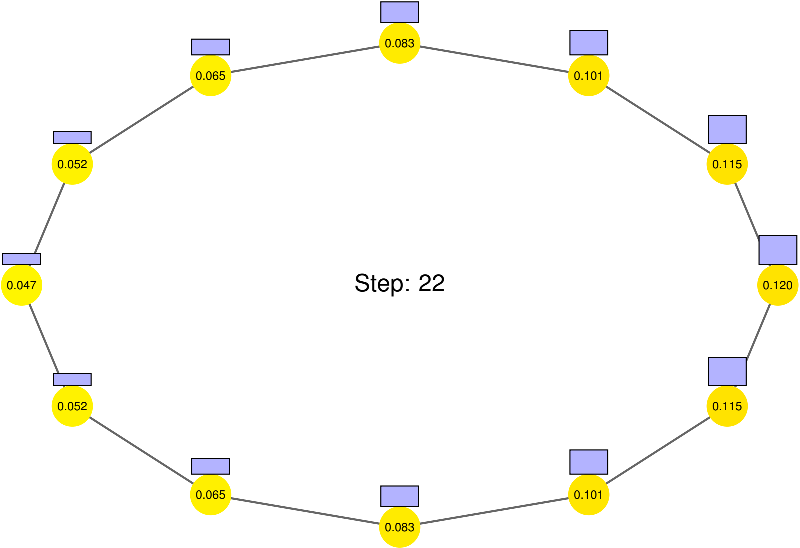

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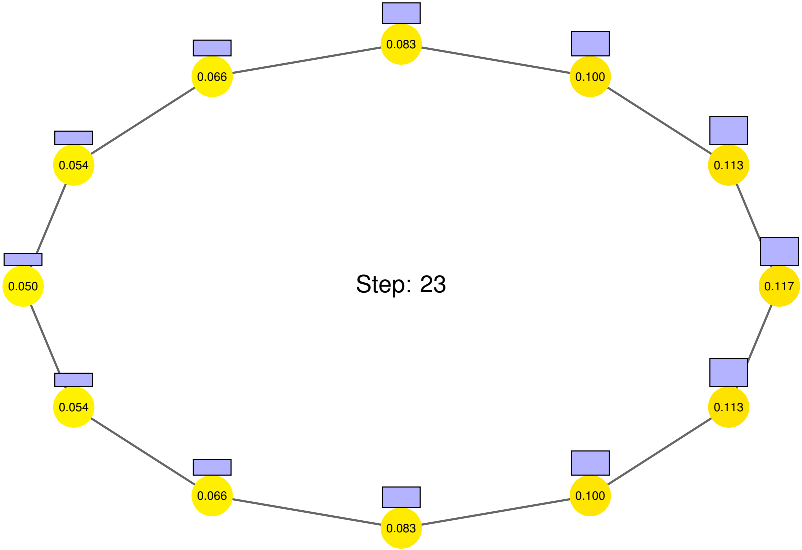

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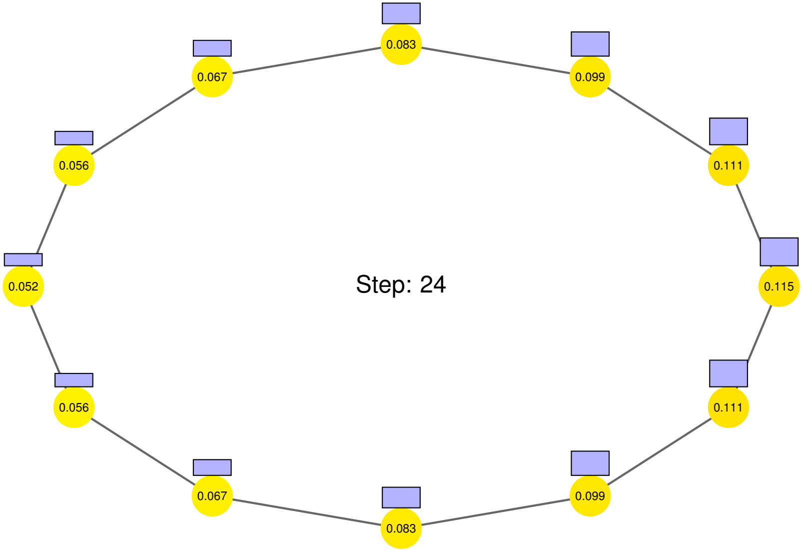

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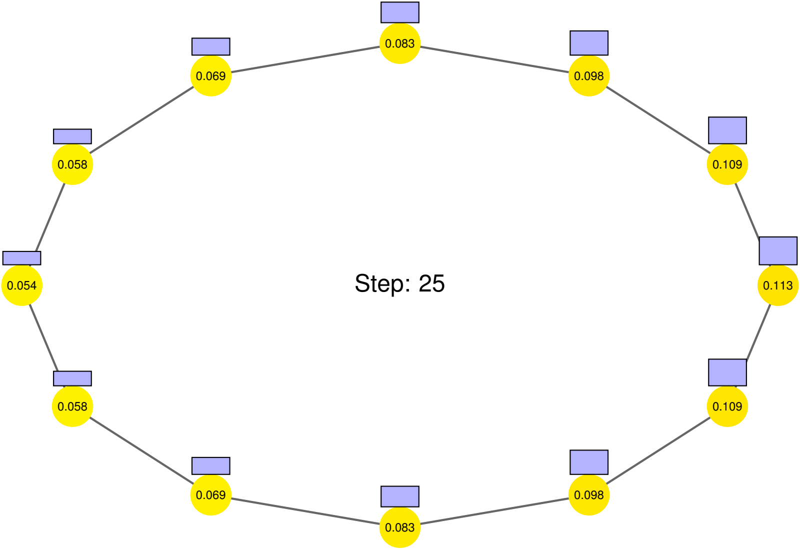

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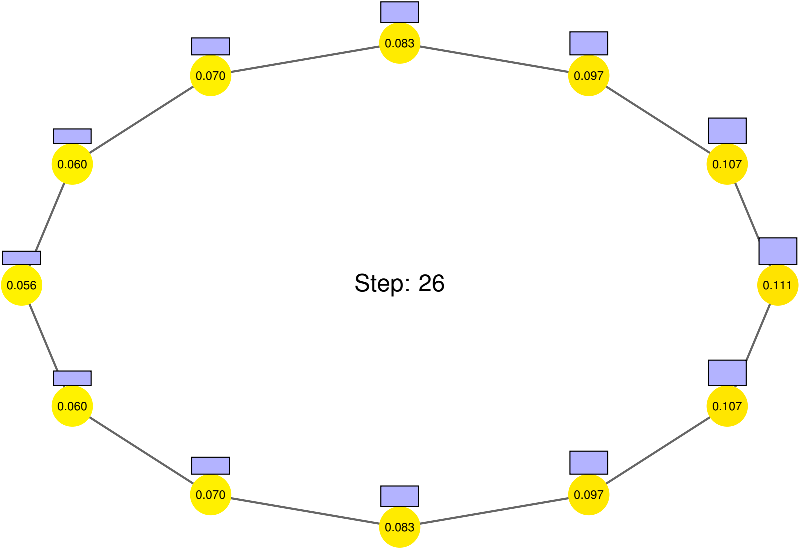

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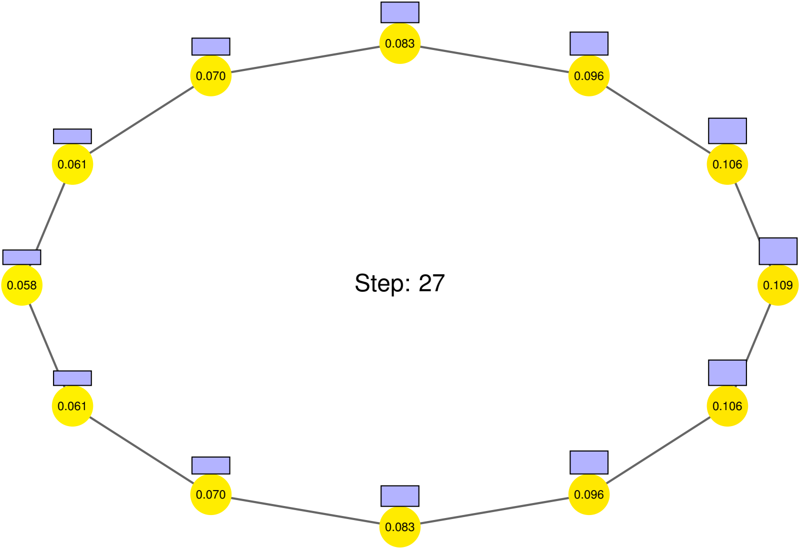

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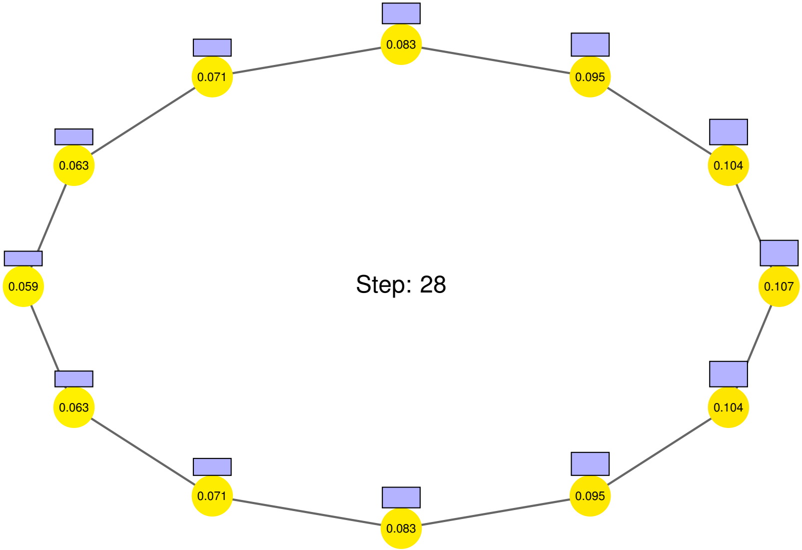

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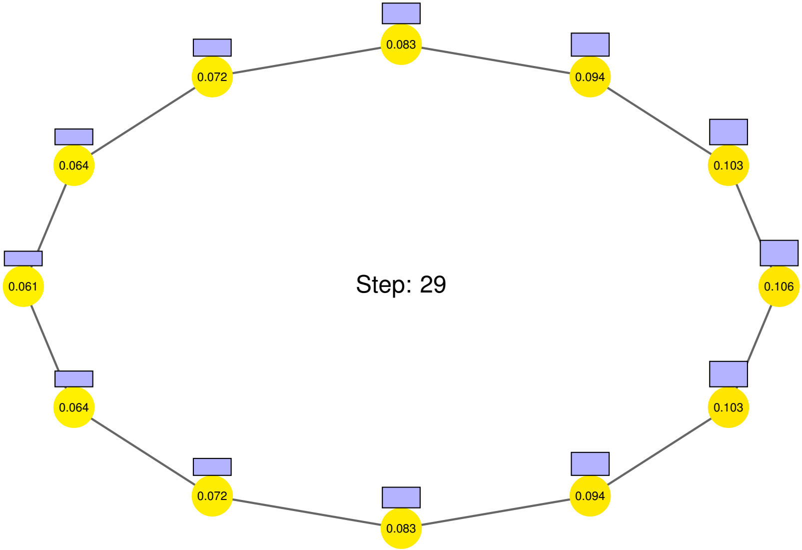

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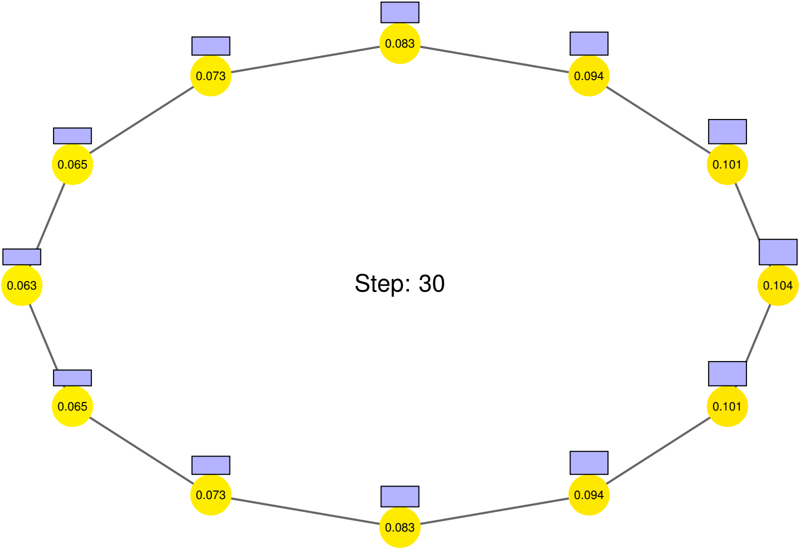

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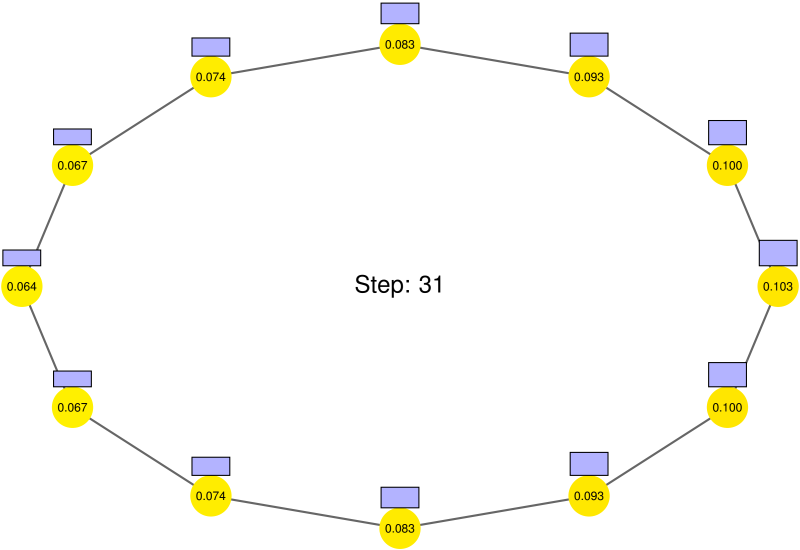

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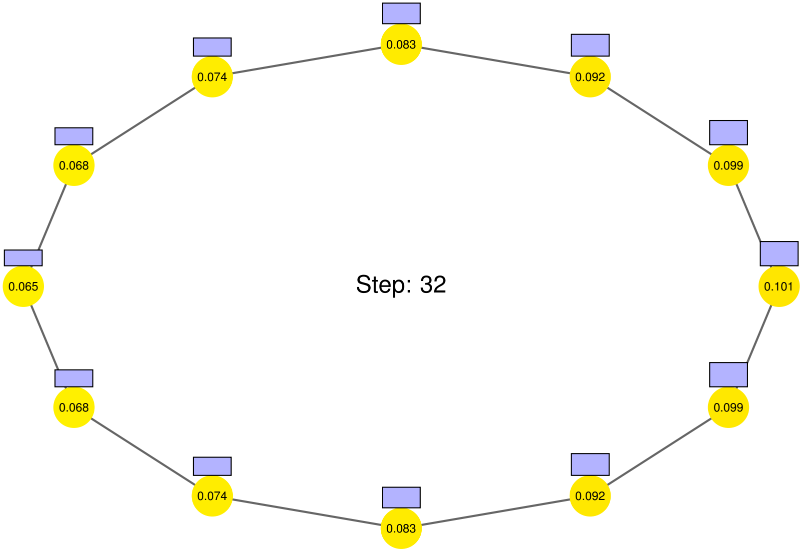

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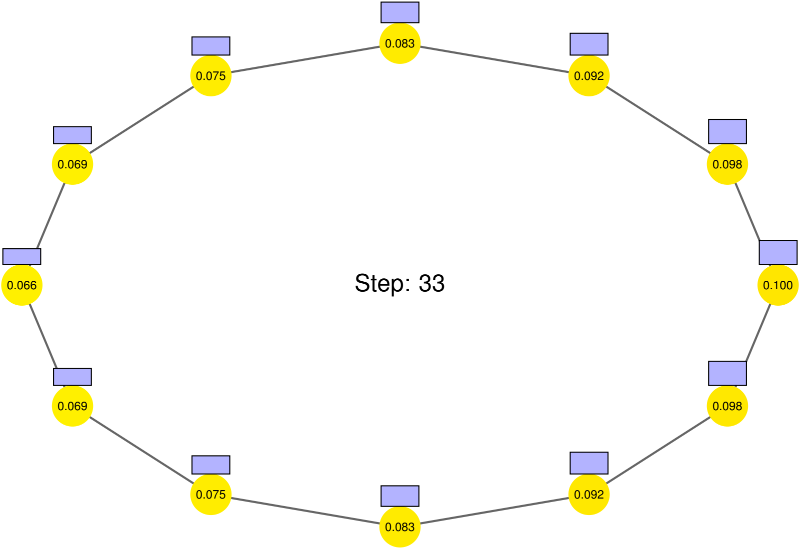

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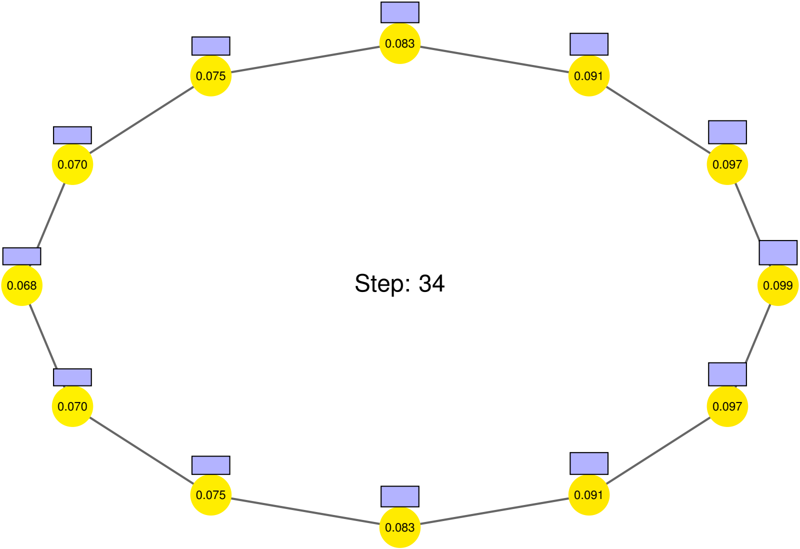

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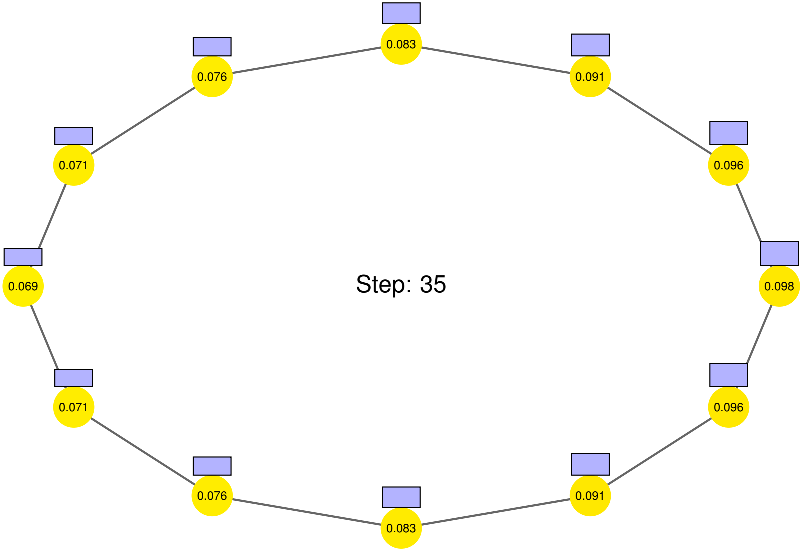

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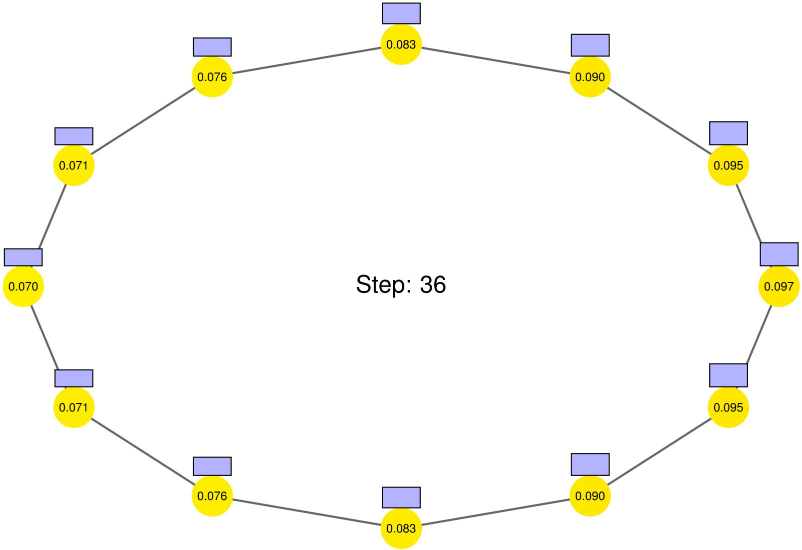

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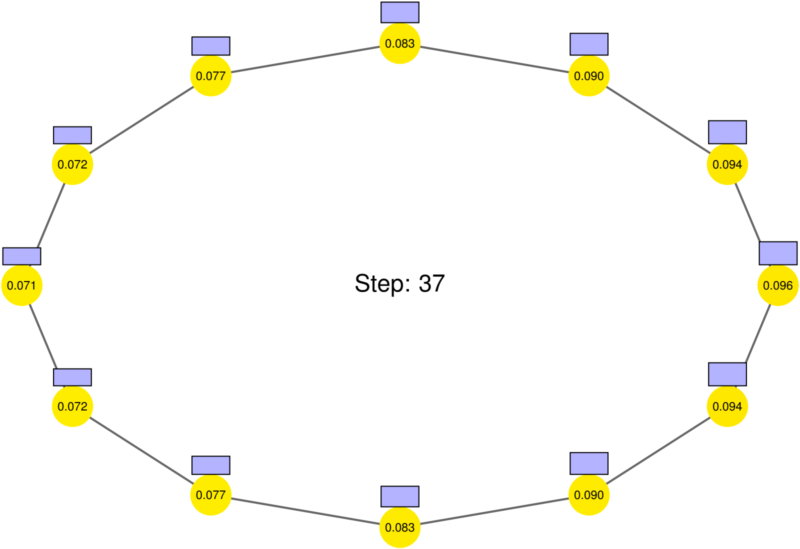

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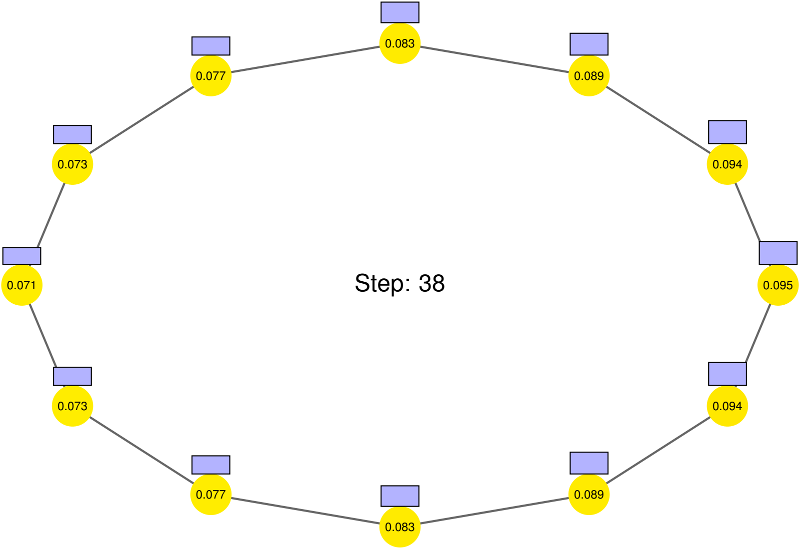

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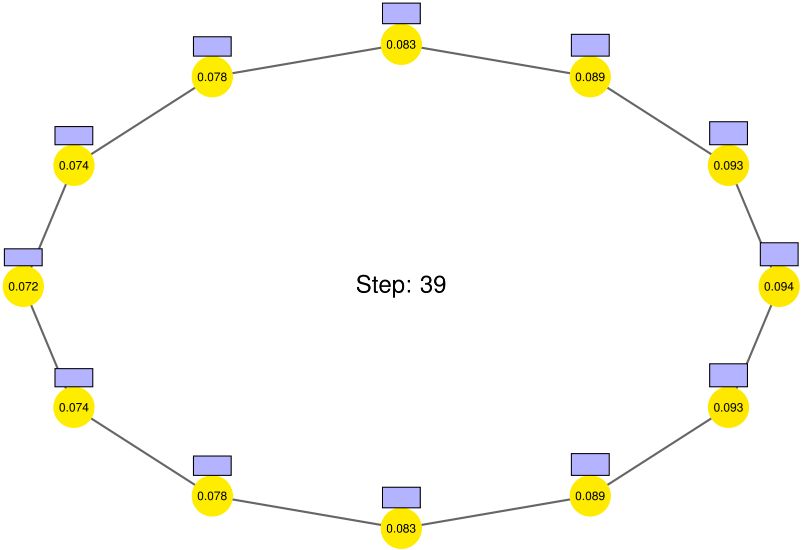

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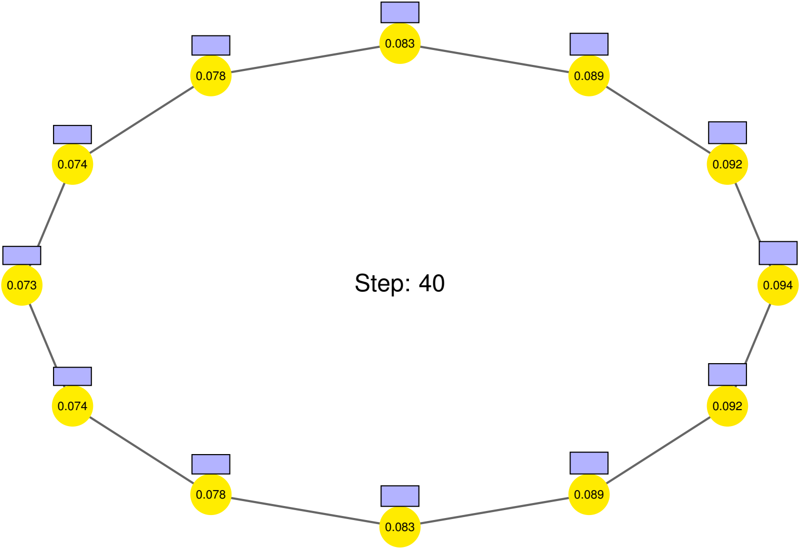

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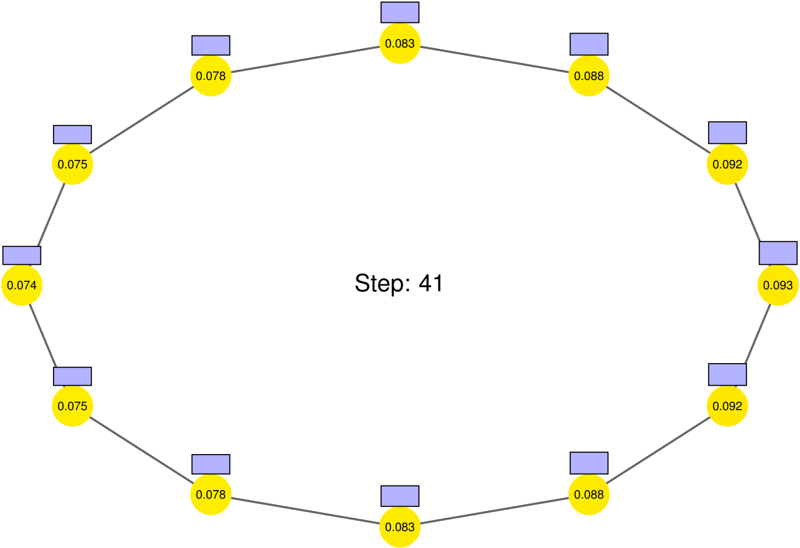

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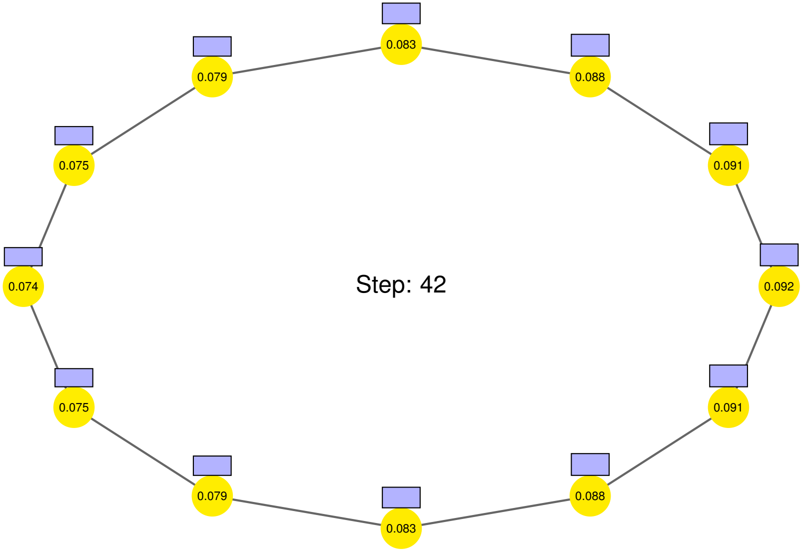

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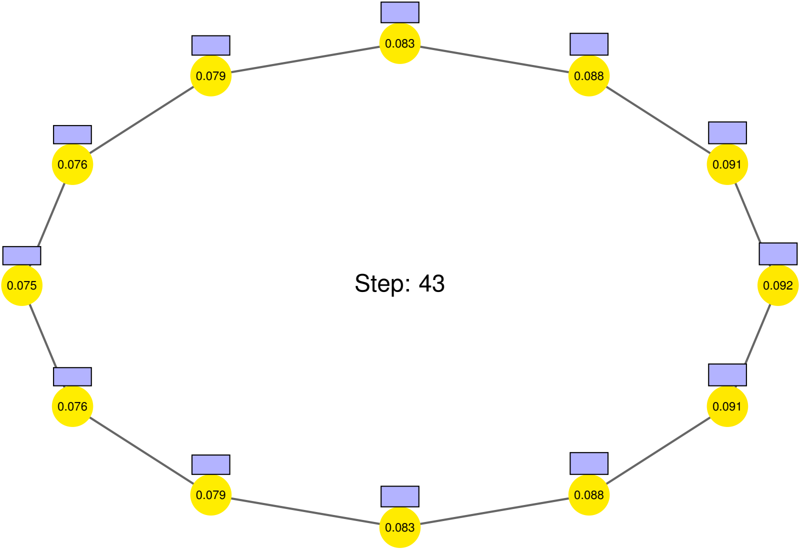

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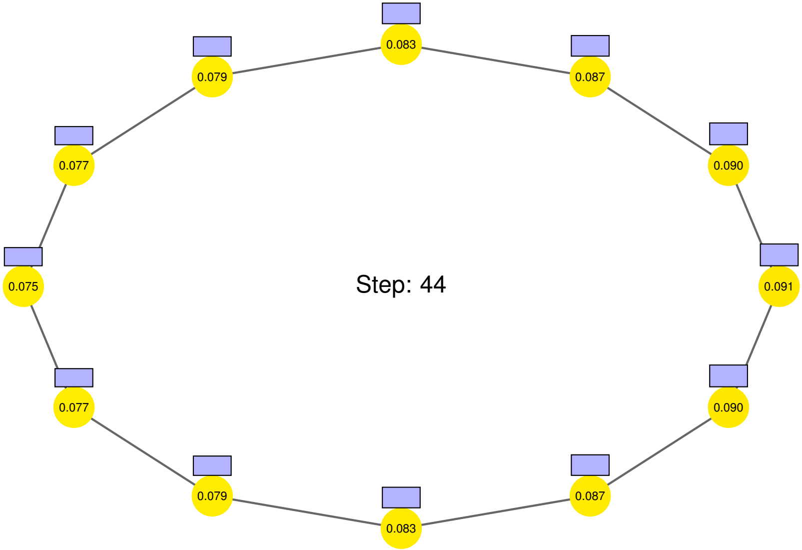

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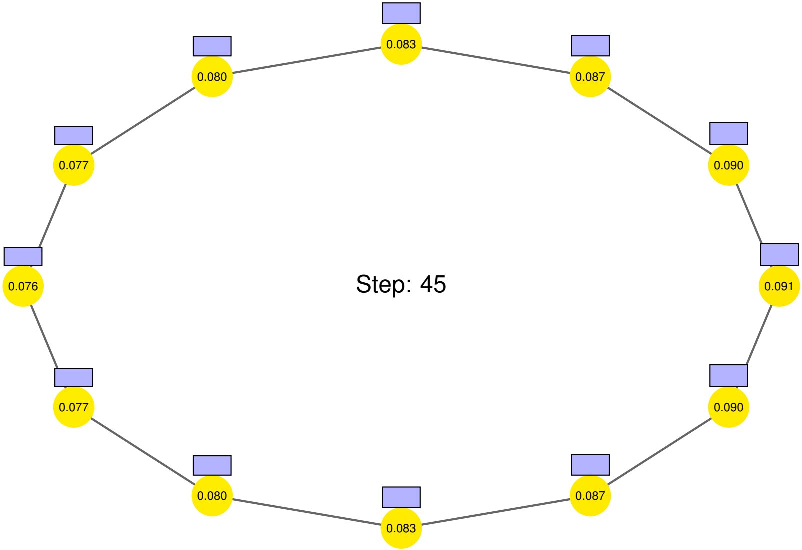

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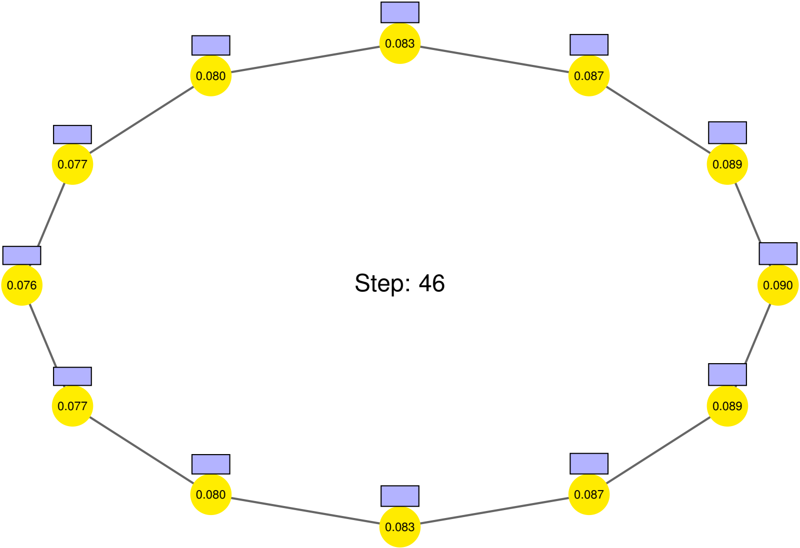

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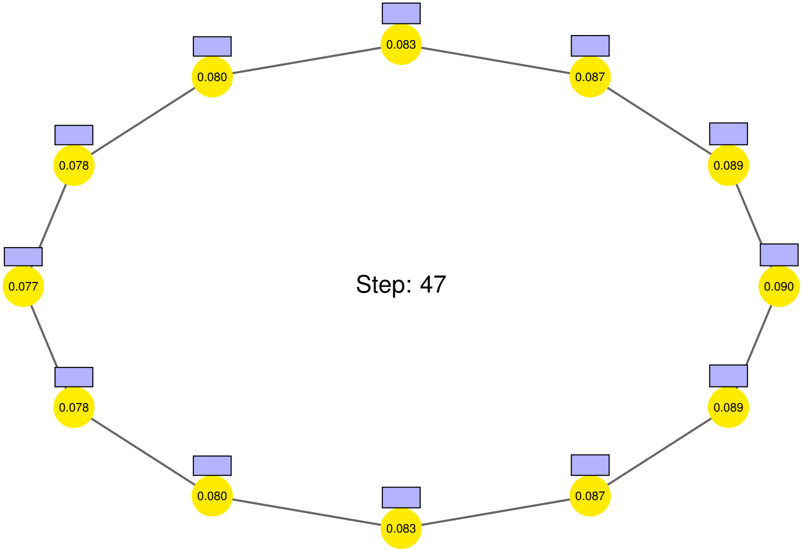

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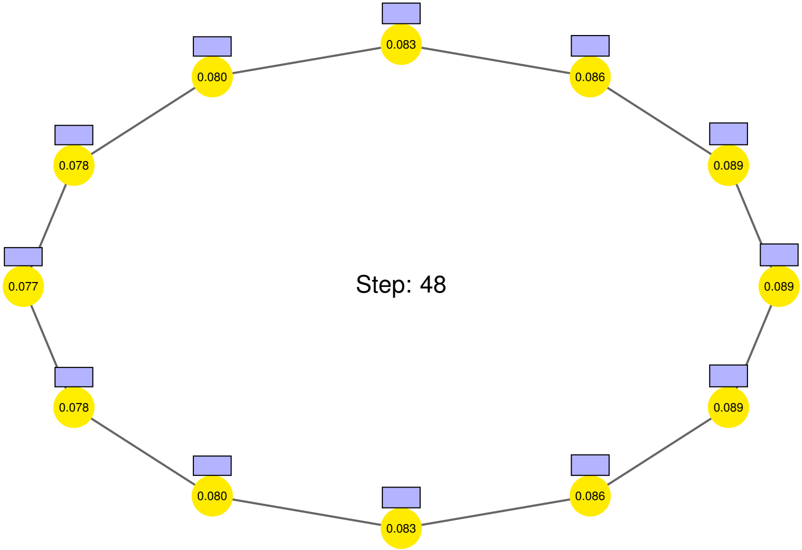

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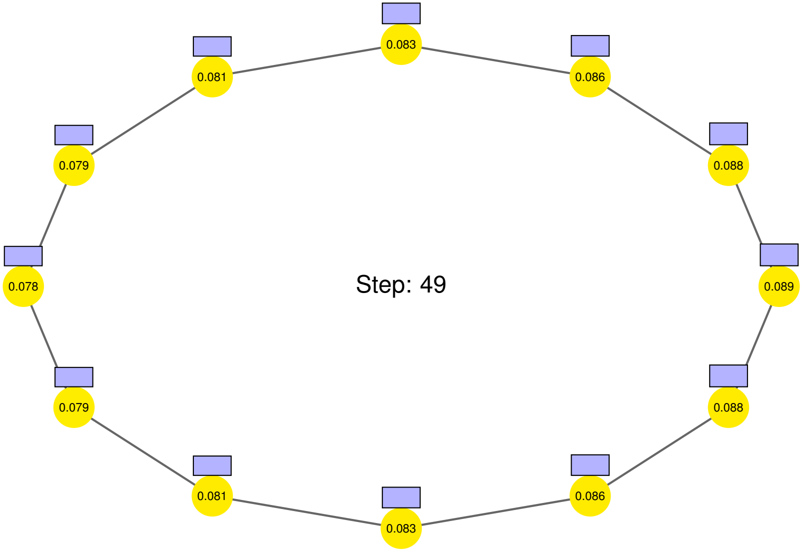

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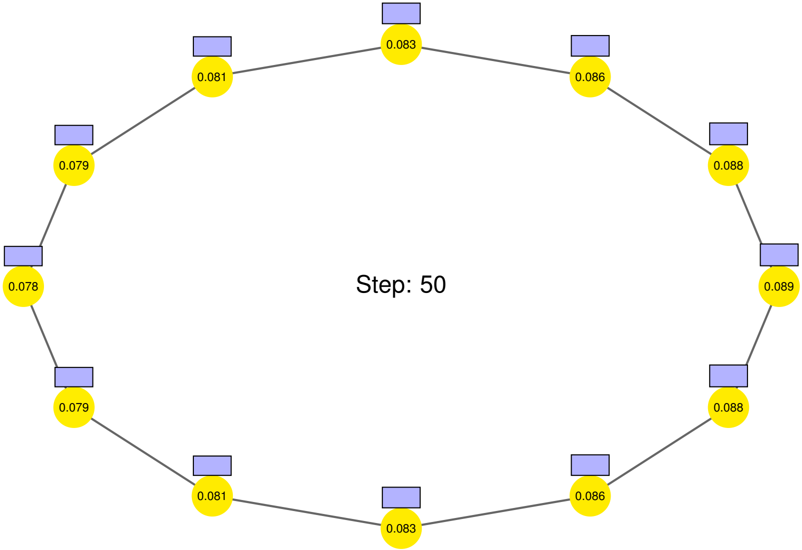

Diffusion of a Random Walk on a Static Cycle

- As long as the probability mass is concentrated on a small set of vertices, substantial progress in the ℓ_2-norm
- More precisely, $\|p^t_{u,\cdot} - \frac{1}{n}\|_2^2 \sim 1/\sqrt{t}$
- This property only requires each graph G^t to be connected (& regular) at each step

Mixing in Dynamic Graphs: Definition

Sequence of (regular) graphs $\mathcal{G} = \{G^{(t)}\}_{t=1}^{\infty}$ on V with transition matrices $\{P^{(t)}\}_{t=1}^{\infty}$

- $\pi P^{(t)} = \pi = 1/n$ for any t

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ℓ_2-mixing time

$$t_{mix}(\mathcal{G}) = \min\left\{ t \;\middle|\; \sum_{y \in V} \left(P_{x,y}^{[0,t]} - \frac{1}{n} \right)^2 \le \frac{1}{10n} \quad \forall x \in V \right\}.$$

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can be extended to non-regular graphs

A Bound on the ℓ_2-Decrease

Key Lemma

Let P be the transition matrix of a random walk on a connected, regular graph $G = (V, E)$. Then for any probability distribution σ,

$$\sum_{u,v \in V} (\sigma(u) - \sigma(v))^2 \cdot P_{u,v} \gtrsim \left(\sum_{u \in V} \left(\sigma(u) - \frac{1}{n} \right)^2 \right)^2 .$$

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As long as $\|\sigma - \frac{1}{n}\|_2^2$ is large $\Rightarrow$ σ is concentrated on a small set of vertices

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$\Rightarrow \exists$ short path between $x^\star = \text{argmax}_x \, \sigma(x)$ and y s.t. $\sigma(y) \ll \sigma(x^\star)$

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$\Rightarrow$ Let ℓ be the length of such path. Then,

$$\sum_{u,v \in V} (\sigma(u) - \sigma(v))^2 P_{u,v} \geq \frac{(\sigma(x^\star) - \sigma(y))^2}{2\ell} \text{ is large} \qquad \square$$

Main Result (covering also non-regular graphs)

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Theorem

Let $\mathcal{G}$ be a sequence of connected graphs of n vertices with unique stationary distribution π. Moreover, denote with $\pi_* = \min_x \pi(x)$. Then:

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- Key Lemma $\Rightarrow$ if ℓ_2-norm is ε, after $O(n/(\pi_*\varepsilon))$ steps it is less than $\varepsilon/2$
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To prove the bound on hitting:

- first obtain a refined bound on the ℓ_2-norm decrease at each step
- relate t-step probabilities to the ℓ_2-norm in variance of the walk
- use probabilistic arguments to relate t-step probabilities to hitting times

Outline

Intro

Random Walks on Sequences of Connected Graphs

Random Walks on Sequences of (Possibly) Disconnected Graphs

Conclusion

What happens when the connectivity properties of the graph change over time?

How to bound mixing when connectivity is intermittent

- In static graphs, the eigenvalues of the individual transition matrices give a good bound on mixing:
$$\frac{1}{1-\lambda} \lesssim t_{mix}(G) \lesssim \frac{\log(n)}{1-\lambda}$$

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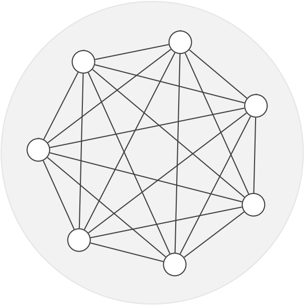

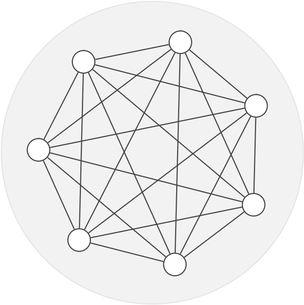

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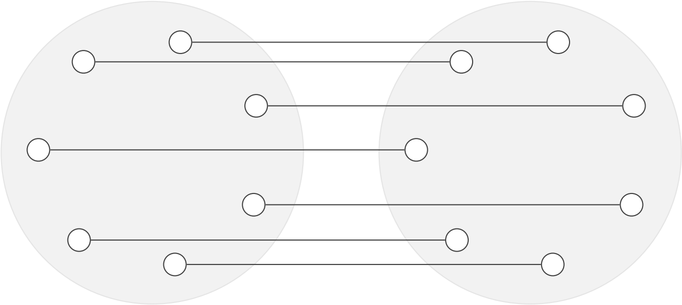

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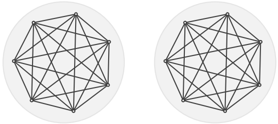

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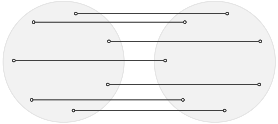

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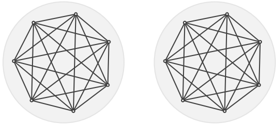

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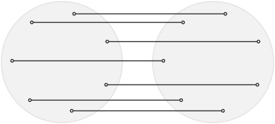

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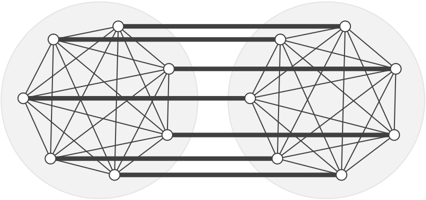

$$1 - \lambda(\overline{P}) = \Omega(1)$$

Mixing based on average connectivity properties

Theorem

Consider a sequence $\mathcal{G}$ with transition matrices $\{P^{(t)}\}_{t=1}^{\infty}$ such that

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2. there exists a time window $T \geq 1$ such that, for any $i \geq 0$, $\overline{P}^{[i \cdot T+1,(i+1) \cdot T]}$ is ergodic with spectral gap greater or equal than $1 - \lambda$

Then, $t_{mix}(\mathcal{G}) = O(T^2 \log(1/\pi_*)/(1-\lambda))$

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Corollary

Suppose that for any time window $\mathcal{I} = [i \cdot T + 1, (i + 1) \cdot T]$ and any subset of vertices $A \subseteq V$ there exists $i \in \mathcal{I}$ such that $\Phi_{P^{(i)}}(A) \geq \phi$. Then,

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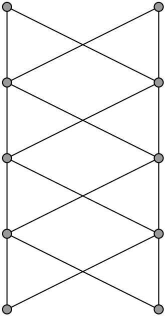
- Why? We can simulate a random walk on a directed graph:

Simulating a Directed Graph using Dynamic Graphs



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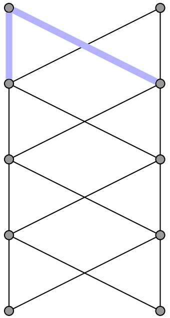
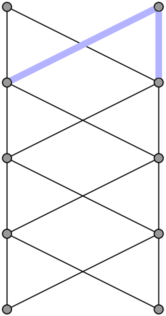


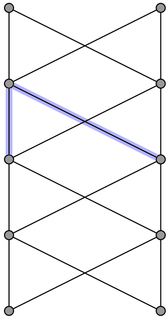
$t = 1$

Simulating a Directed Graph using Dynamic Graphs



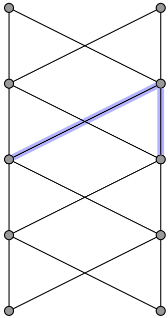
$t = 2$

Simulating a Directed Graph using Dynamic Graphs



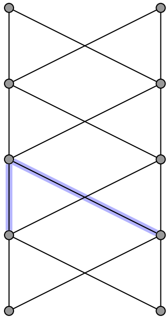
$t = 3$

Simulating a Directed Graph using Dynamic Graphs



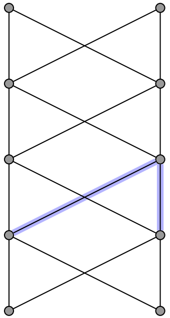
$t = 4$

Simulating a Directed Graph using Dynamic Graphs



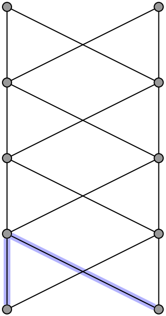
$t = 5$

Simulating a Directed Graph using Dynamic Graphs



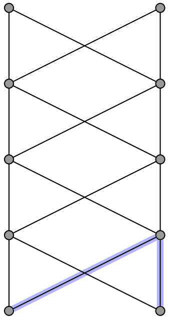
$t = 6$

Simulating a Directed Graph using Dynamic Graphs



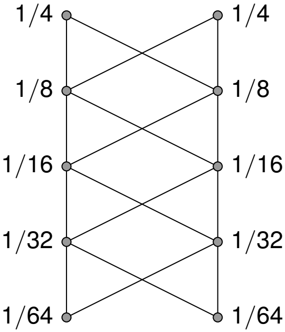
$t = 7$

Simulating a Directed Graph using Dynamic Graphs



$t = 8$

Simulating a Directed Graph using Dynamic Graphs



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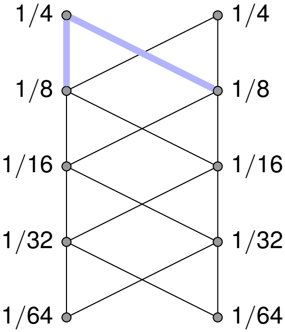

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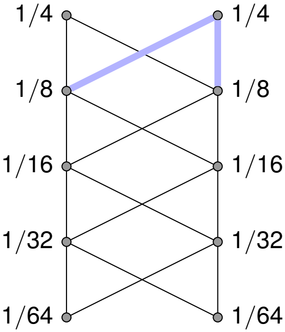

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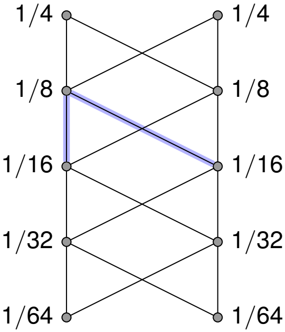
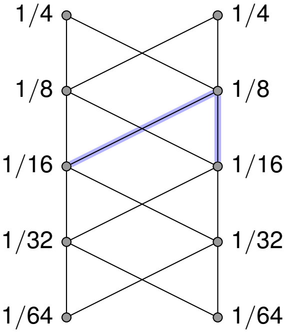


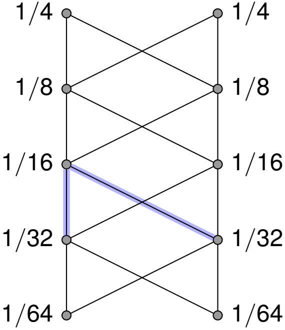
$t = 3$

Simulating a Directed Graph using Dynamic Graphs



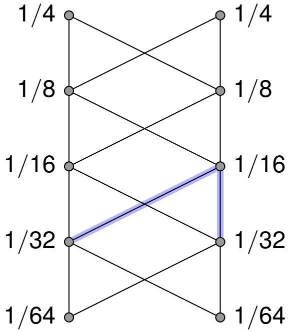
$t = 4$

Simulating a Directed Graph using Dynamic Graphs



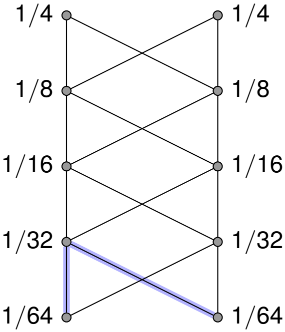
$t = 5$

Simulating a Directed Graph using Dynamic Graphs



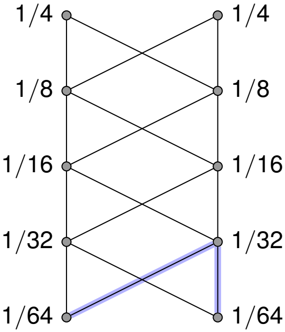
$t = 6$

Simulating a Directed Graph using Dynamic Graphs



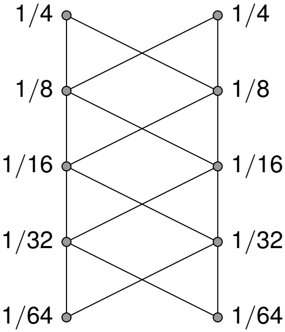
$t = 7$

Simulating a Directed Graph using Dynamic Graphs



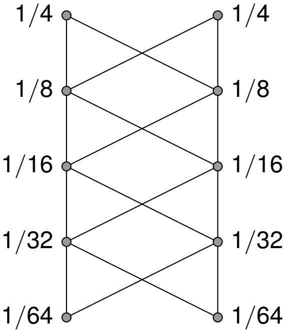
$t = 8$

Simulating a Directed Graph using Dynamic Graphs



Random Walk Behaviour:

Simulating a Directed Graph using Dynamic Graphs



Random Walk Behaviour:

- Since the stationary distribution is exponentially small for the vertices at the bottom, hitting time is exponential in n
- However, average transition matrix $\overline{P}$ can be easily made ergodic (add same cycle of $n-2$ matrices in reverse order)
- $\Rightarrow$ mixing time polynomial in n by our theorem!

Outline

Intro

Random Walks on Sequences of Connected Graphs

Random Walks on Sequences of (Possibly) Disconnected Graphs

Conclusion

Conclusions and Future Work

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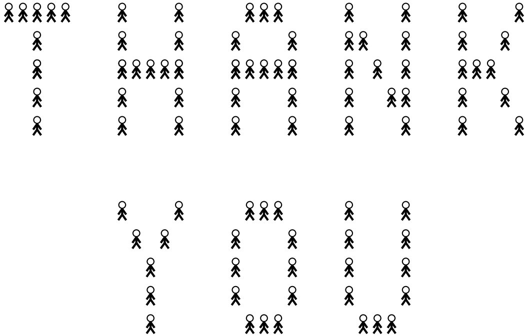
But: In real-world graphs, also the vertex set may change!

The End



The End