Locality via Alternation?

Fabian Reiter

LIGM, Université Gustave Eiffel

ADGA 2024

Two characterizations of NP

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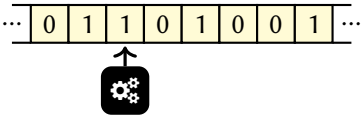
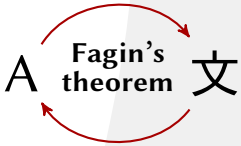


Nondet. polynomial-time
Turing machines



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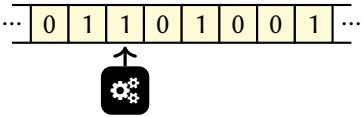

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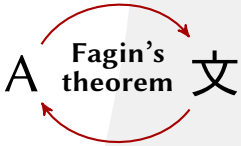

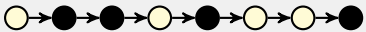

$\exists R\colon \mathit{Bijective}(R) \wedge \forall x, y\colon R(x, y) \to (\bullet x \leftrightarrow \neg \bullet y)$

Existential fragment of
second-order logic

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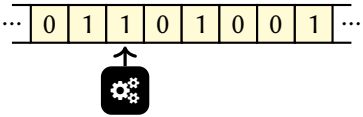

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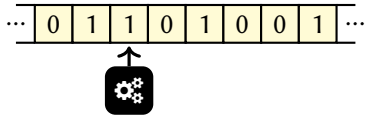

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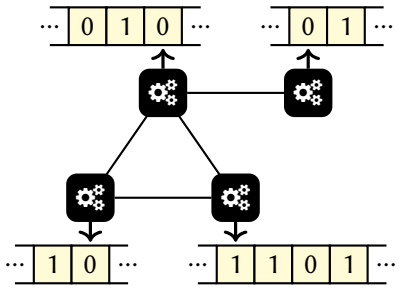

A **Fagin's theorem ++** 文

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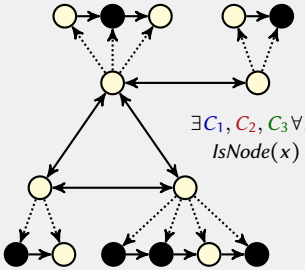

$\exists C_1, C_2, C_3 \forall x\colon$
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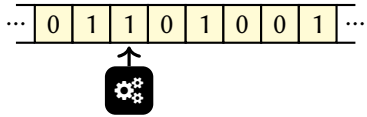

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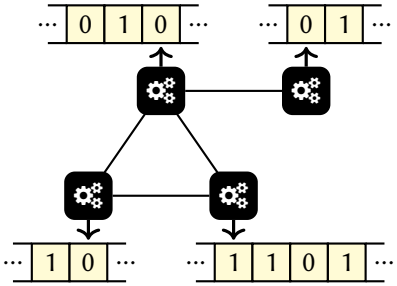

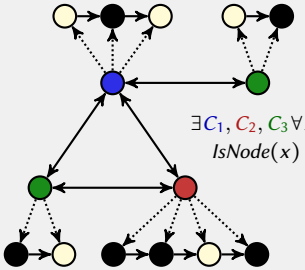

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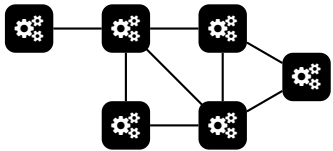
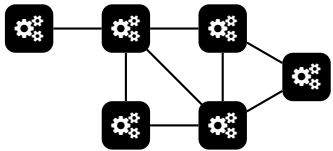
Model of computation



The LOCAL model

- Network of nodes with IDs & labels
- Same algorithm on all nodes
- Synchronous communication rounds

Model of computation



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- Constant number of rounds
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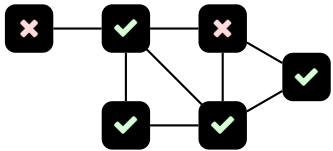

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(some nodes of odd degree)

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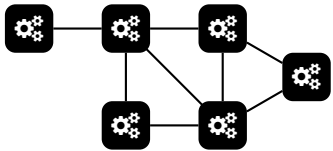

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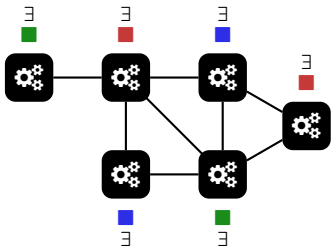

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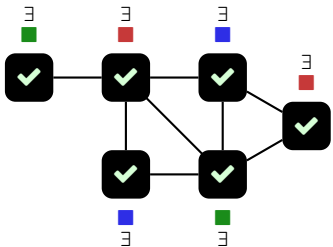

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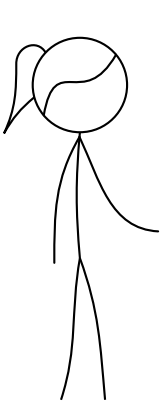

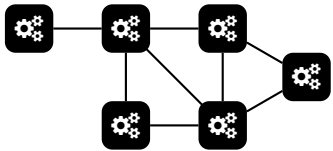

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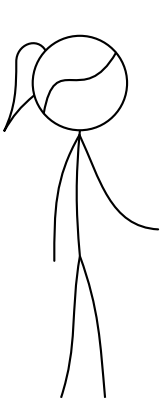

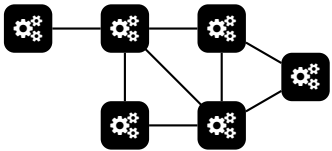

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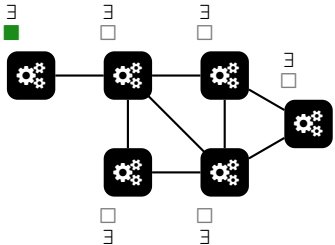

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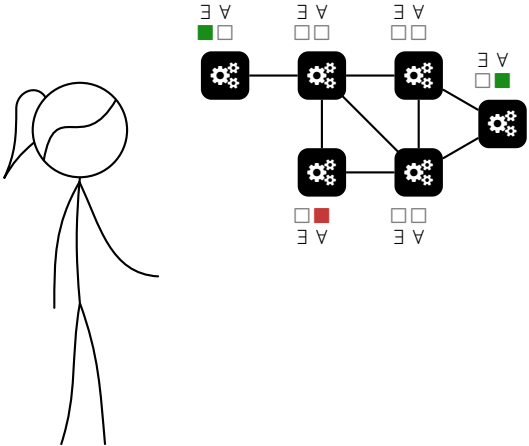

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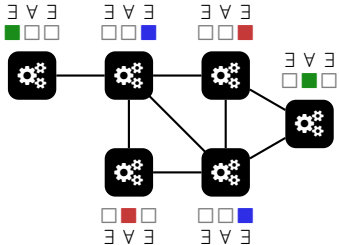

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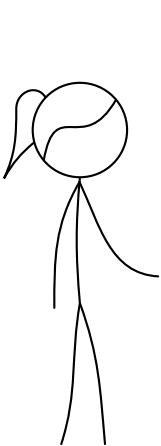

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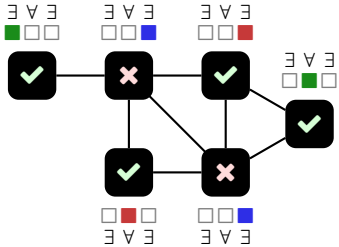

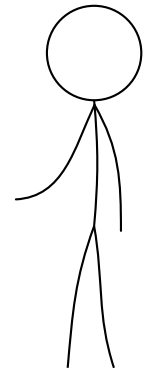

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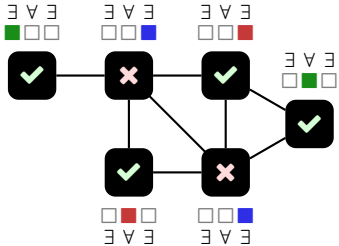

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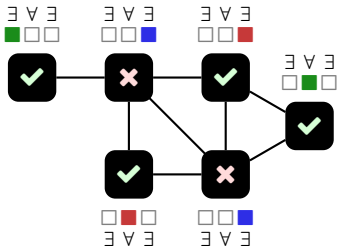

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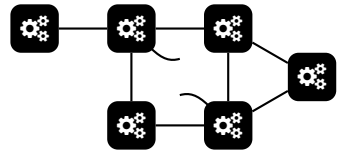

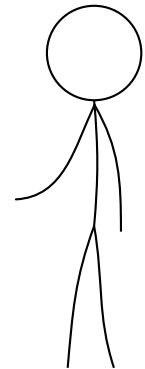

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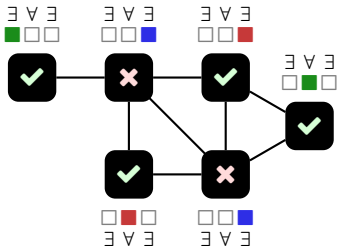

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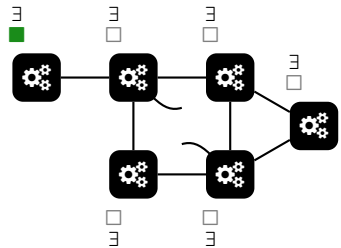

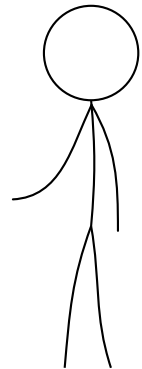

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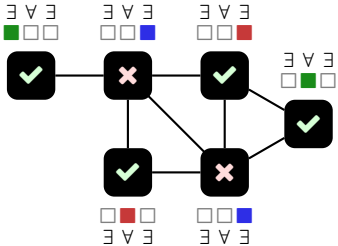

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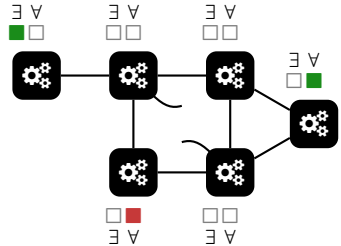

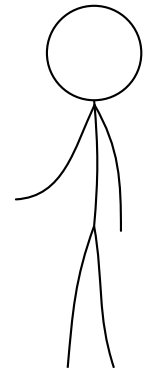

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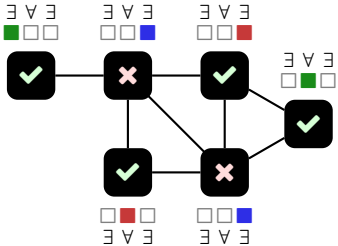

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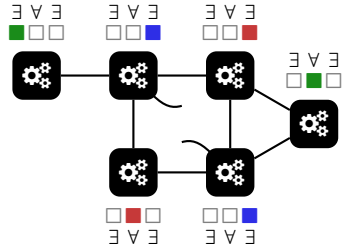

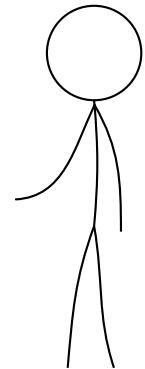

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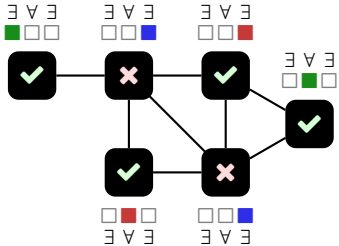

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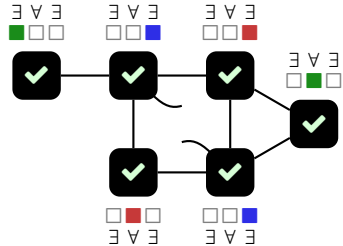

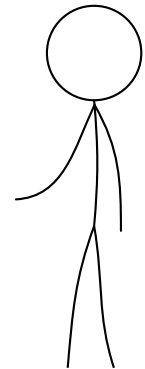

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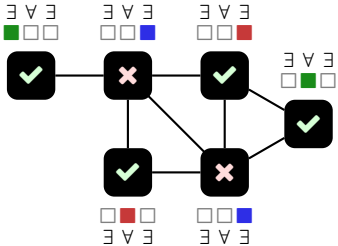

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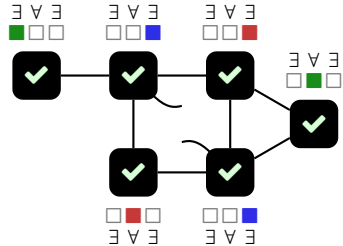

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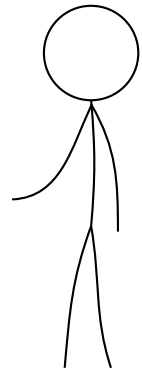

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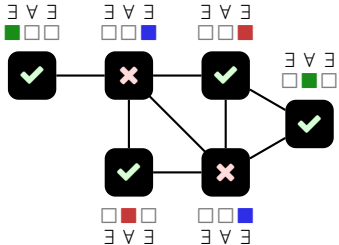

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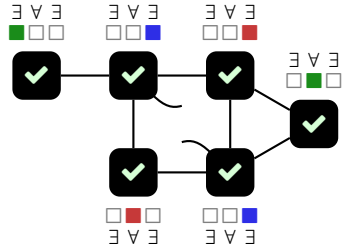

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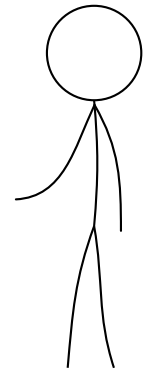

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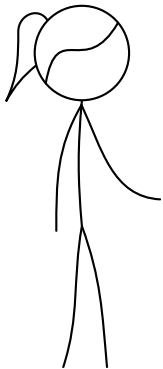
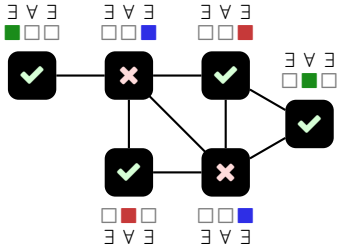
$\mathbf{\Sigma}_3$ $\exists\,\forall\,\exists$

$\mathbf{\Sigma}_2$ $\exists\,\forall$

$\mathbf{\Sigma}_1$ $\exists$

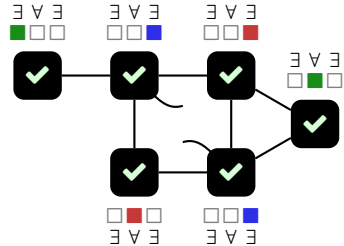
Alternation





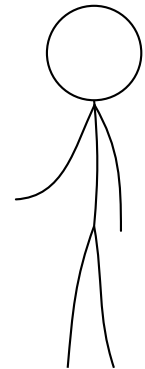
not 3-round 3-colorable

(Adam has a winning strategy)



3-round 3-colorable

(Eve has a winning strategy)



Eve (∃)

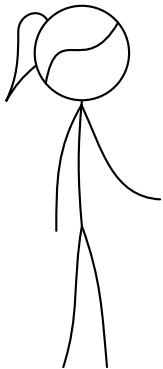
Adam (∀)

$\longrightarrow$ $\mathbf{\Sigma}_3$ $\exists\,\forall\,\exists$

$\mathbf{\Sigma}_2$ $\exists\,\forall$

$\mathbf{\Sigma}_1$ $\exists$

Alternation



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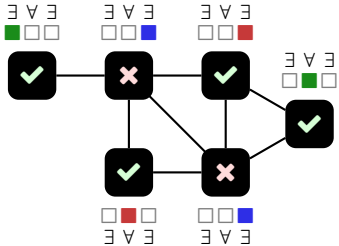

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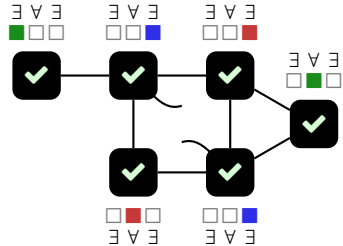

3-round 3-colorable

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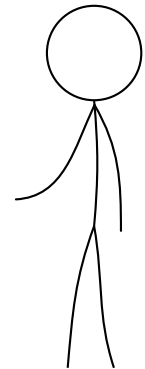


Adam (∀)

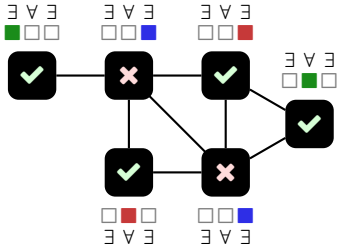
$\longrightarrow \quad \mathbf{\Sigma}_3 \quad \exists\,\forall\,\exists$

$\quad\quad\;\; \mathbf{\Sigma}_2 \quad \exists\,\forall$

$\longrightarrow \quad \mathbf{\Sigma}_1 \quad \exists$

Alternation

Eve (∃)



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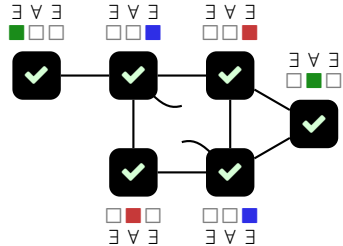


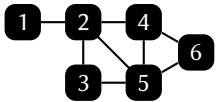
3-round 3-colorable

(Eve has a winning strategy)

Adam (∀)

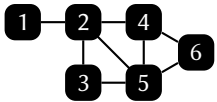
⟶	$\mathbf{\Sigma}_3$	$\exists\,\forall\,\exists$	$\mathbf{\Pi}_3$	$\forall\,\exists\,\forall$
	$\mathbf{\Sigma}_2$	$\exists\,\forall$	$\mathbf{\Pi}_2$	$\forall\,\exists$
⟶	$\mathbf{\Sigma}_1$	$\exists$	$\mathbf{\Pi}_1$	$\forall$

Related work



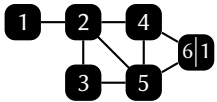
Feuilloley Fraigniaud Hirvonen (ICALP 2016)	Balliu D'Angelo Fraigniaud Olivetti (STACS 2017)	Aldema Tshuva Oshman (PODC 2022)	This work

Related work



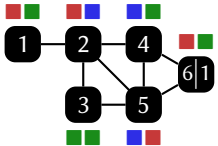
	Feuilloley Fraigniaud Hirvonen (ICALP 2016)	Balliu D'Angelo Fraigniaud Olivetti (STACS 2017)	Aldema Tshuva Oshman (PODC 2022)	This work
ID uniqueness	global	global	global	

Related work



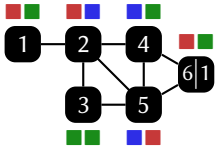
	Feuilloley Fraigniaud Hirvonen (ICALP 2016)	Balliu D'Angelo Fraigniaud Olivetti (STACS 2017)	Aldema Tshuva Oshman (PODC 2022)	This work
ID uniqueness	global	global	global	local

Related work



	Feuilloley Fraigniaud Hirvonen (ICALP 2016)	Balliu D'Angelo Fraigniaud Olivetti (STACS 2017)	Aldema Tshuva Oshman (PODC 2022)	This work
ID uniqueness	global	global	global	local
IDs in certificates	yes	no	no	

Related work



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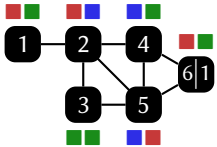
ID uniqueness	global	global	global	local
IDs in certificates	yes	no	no	(yes)

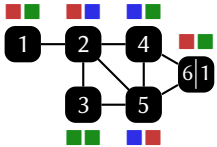
Related work



	Feuilloley Fraigniaud Hirvonen (ICALP 2016)	Balliu D'Angelo Fraigniaud Olivetti (STACS 2017)	Aldema Tshuva Oshman (PODC 2022)	This work
ID uniqueness	global	global	global	local
IDs in certificates	yes	no	no	(yes)
Certificate size	$O(\log n)$	unbounded	poly n	

n: number of nodes

Related work

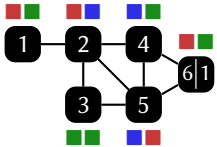


	Feuilloley Fraigniaud Hirvonen (ICALP 2016)	Balliu D'Angelo Fraigniaud Olivetti (STACS 2017)	Aldema Tshuva Oshman (PODC 2022)	This work
ID uniqueness	global	global	global	local
IDs in certificates	yes	no	no	(yes)
Certificate size	$O(\log n)$	unbounded	poly n	poly $\lvert N_r(v)\rvert$

n: number of nodes

$\lvert N_r(v)\rvert$: size of node v's r-neighborhood

Related work

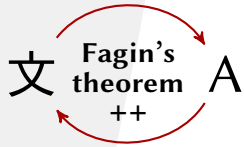


	Feuilloley Fraigniaud Hirvonen (ICALP 2016)	Balliu D'Angelo Fraigniaud Olivetti (STACS 2017)	Aldema Tshuva Oshman (PODC 2022)	This work
ID uniqueness	global	global	global	local
IDs in certificates	yes	no	no	(yes)
Certificate size	$O(\log n)$	unbounded	poly n	poly $\lvert N_r(v)\rvert$
Computation time	unbounded	unbounded	poly n	poly $\lvert N_r(v)\rvert$

n: number of nodes

$|N_r(v)|$: size of node v's r-neighborhood

Using logic and automata theory



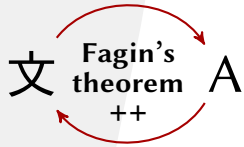
The LOCAL model

+ locally unique IDs
+ local-polynomial bounds

Using logic and automata theory

Monadic second-order logic (MSO)

- *Yields an infinite hierarchy on grids* [1].
- *Satisfies a locality property* [2].



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[1] Matz, Schweikardt, Thomas (2002)
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Using logic and automata theory

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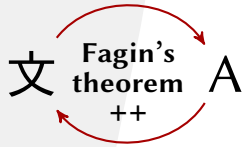
Finite-state automata

- *Satisfy a pumping lemma* [3].
- *Are equivalent to MSO on words* [4].



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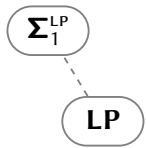
[4] Büchi (1960) & Elgot (1961) & Trakhtenbrot (1962)

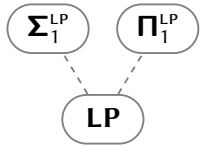
The local-polynomial hierarchy

LP

The local-polynomial hierarchy



The local-polynomial hierarchy



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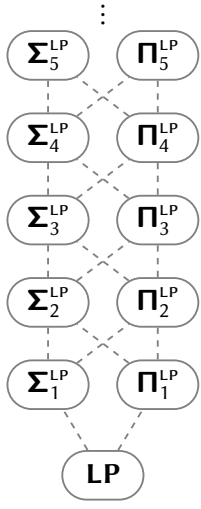

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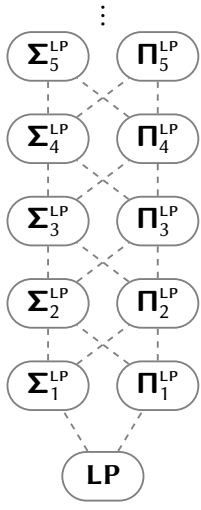
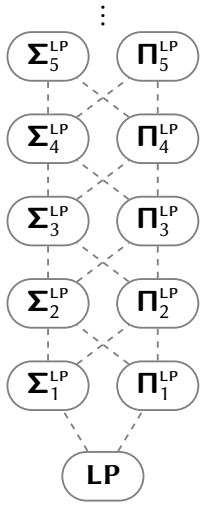


Connection to classical complexity:

$$\Sigma_\ell^{\mathrm{P}} = \Sigma_\ell^{\mathrm{LP}}|_{\mathrm{NODE}} \qquad \Pi_\ell^{\mathrm{P}} = \Pi_\ell^{\mathrm{LP}}|_{\mathrm{NODE}}$$

The local-polynomial hierarchy



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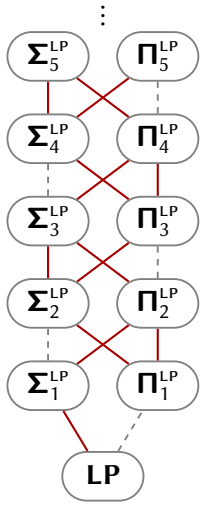
The local-polynomial hierarchy



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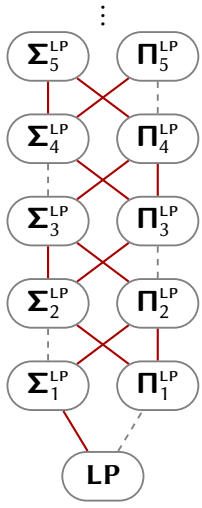
Theorem: — Strict inclusions

The local-polynomial hierarchy



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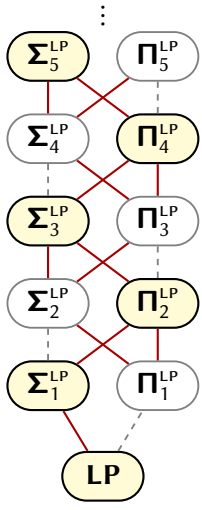
THEOREM: — Strict inclusions
--- Equalities iff $\mathbf{P} = \mathbf{NP}$

The local-polynomial hierarchy



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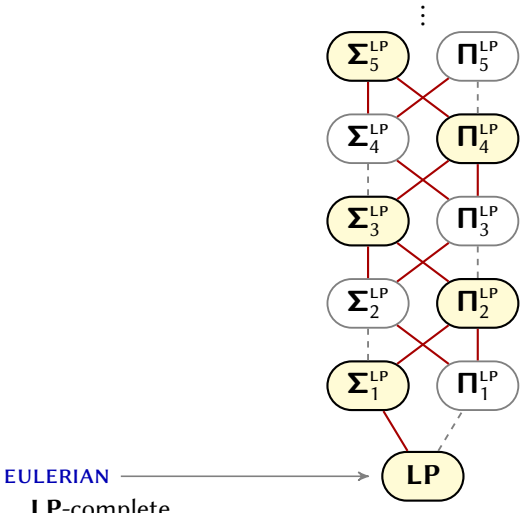

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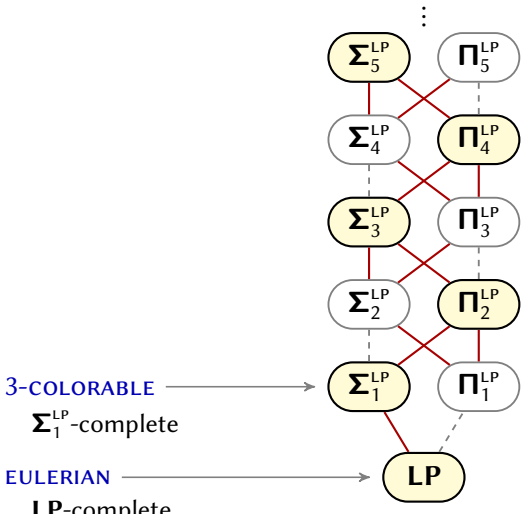

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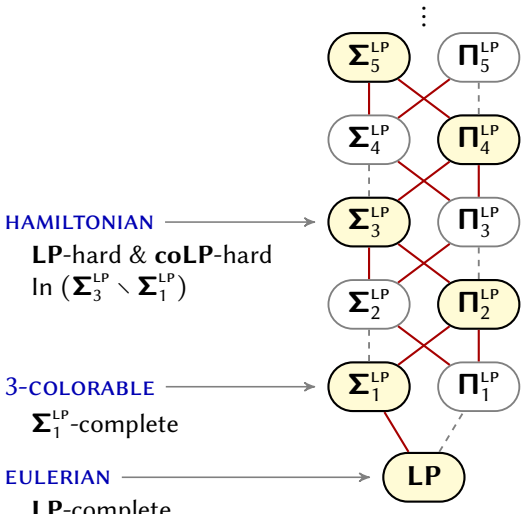

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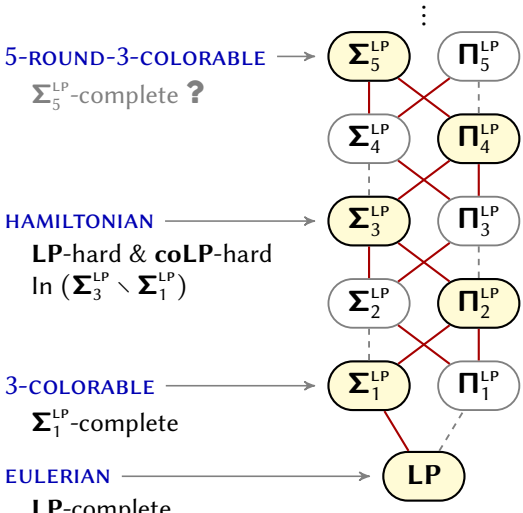

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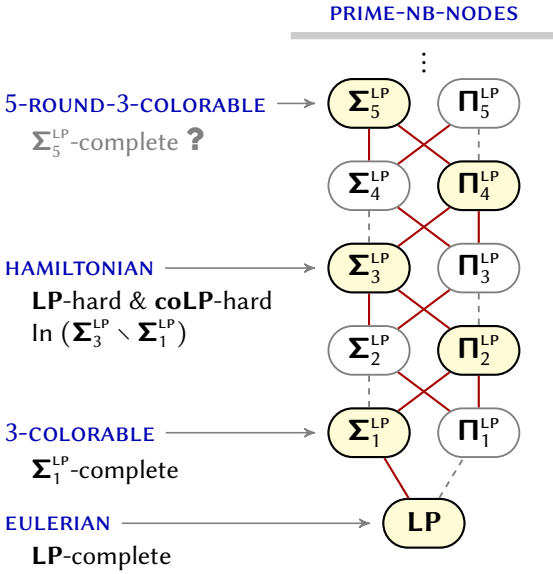
The local-polynomial hierarchy



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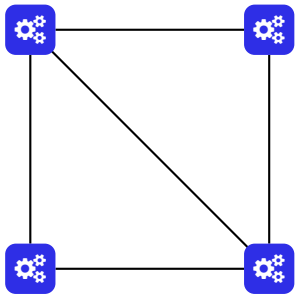
A local-polynomial reduction

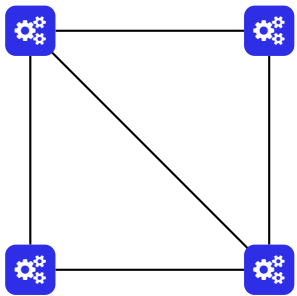
G:



All nodes of G are blue

A local-polynomial reduction

G:



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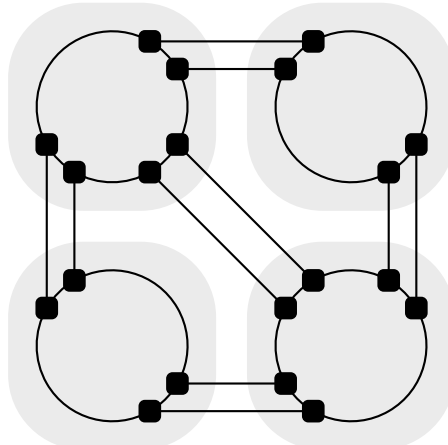
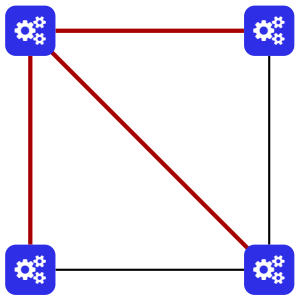


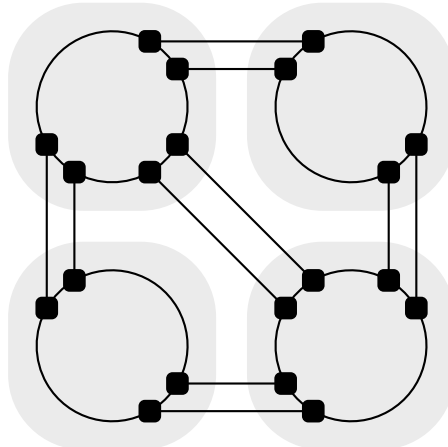
All nodes of G are blue $\iff$ G' contains a Hamiltonian cycle

A local-polynomial reduction

G:



G':



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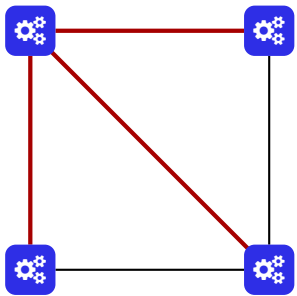
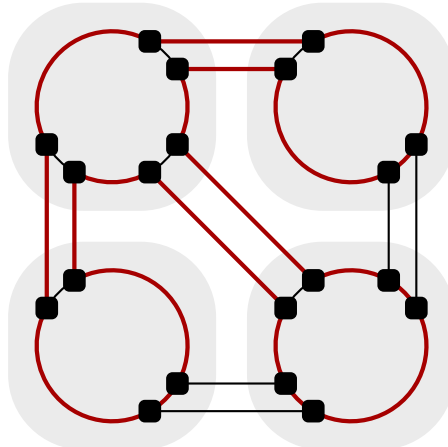


G':



All nodes of G are blue $\iff$ G' contains a Hamiltonian cycle

A local-polynomial reduction

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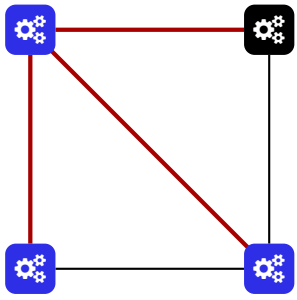

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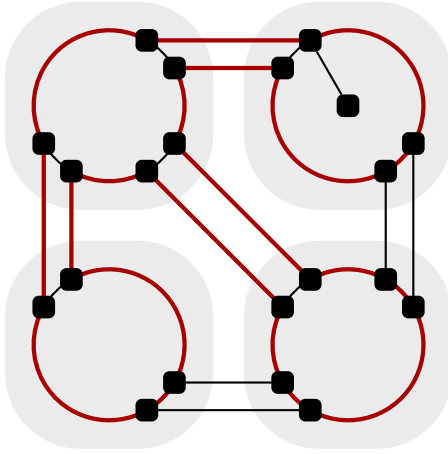

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Extending a classical reduction

G is satisfiable $\iff$ G' is 3-colorable

G: $P_1 \vee \bar{P}_2 \vee \bar{P}_3$

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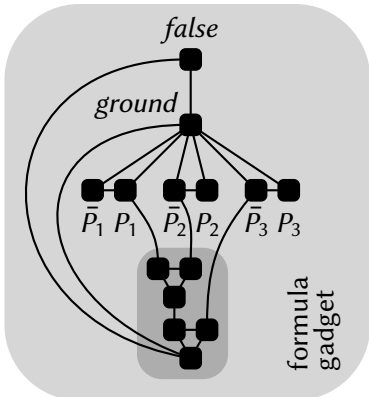

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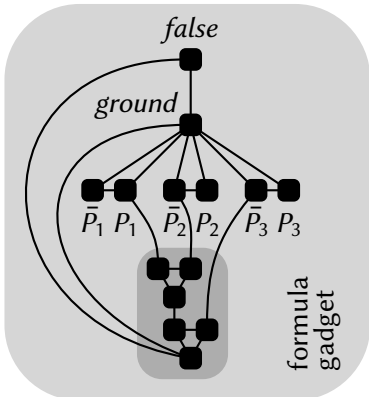

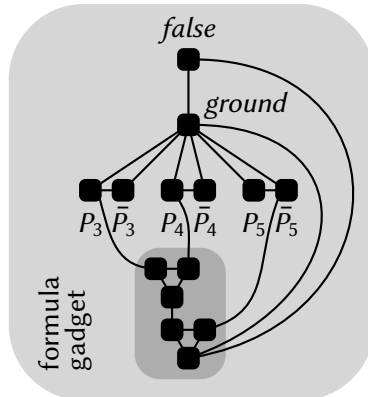

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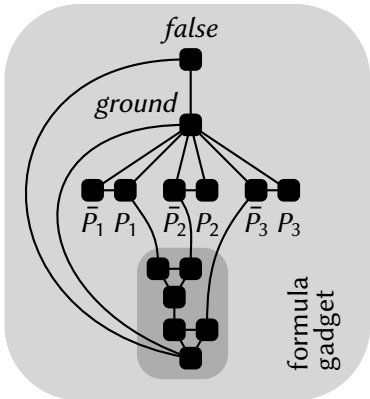

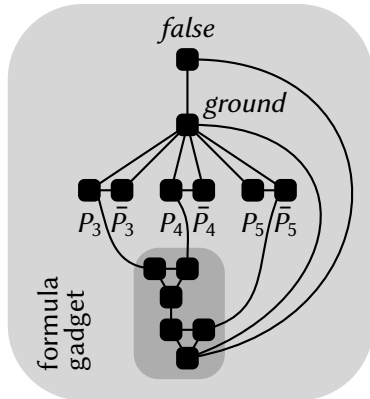

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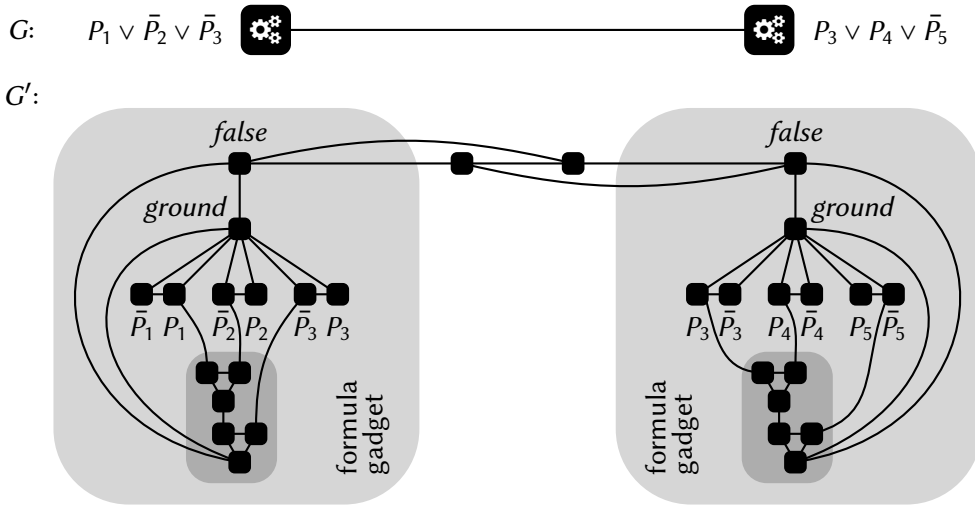

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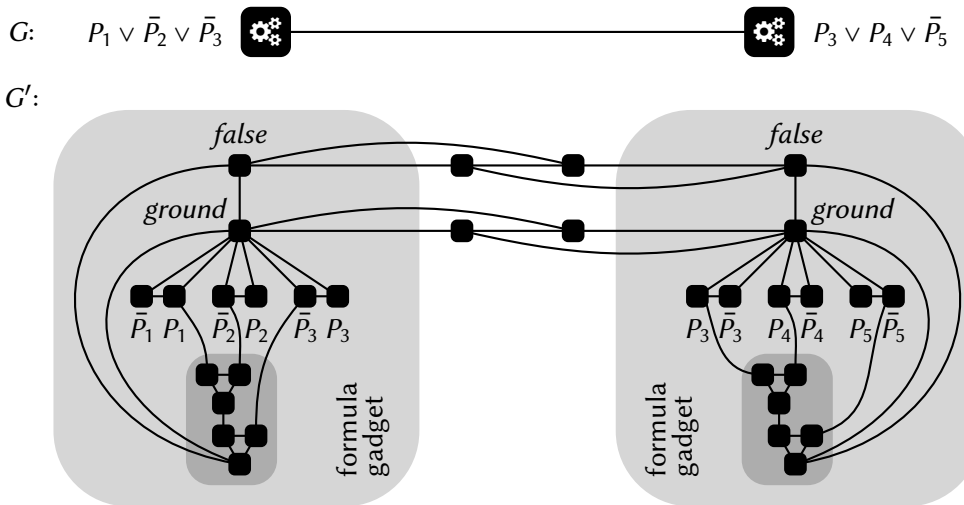

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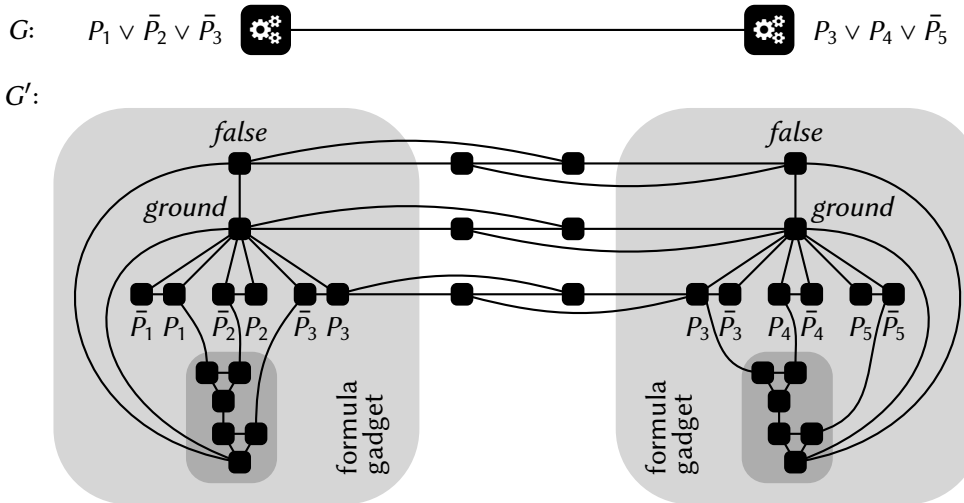

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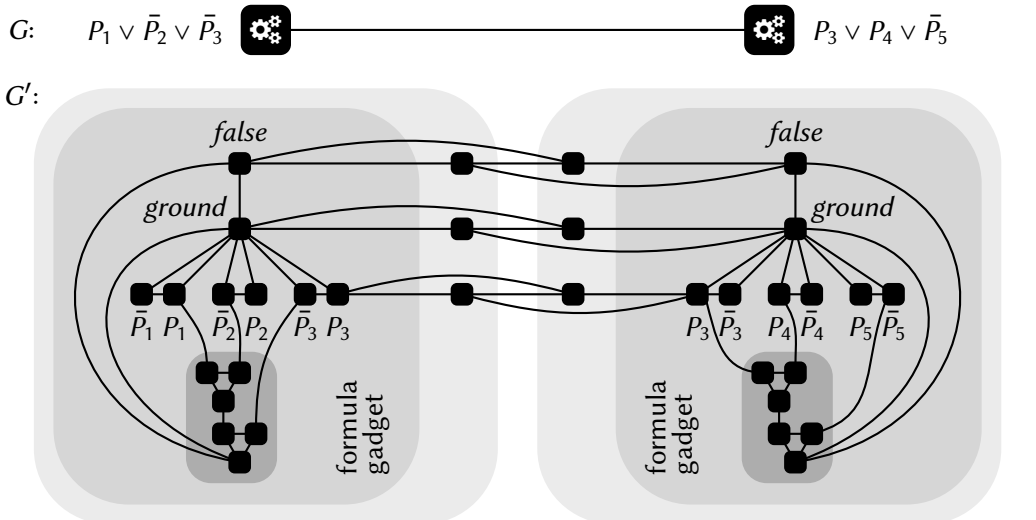
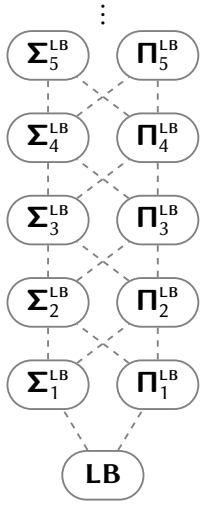


The local-*bounded* hierarchy

LP-hierarchy ⟶ **LB**-hierarchy

polynomial bounds — arbitrary **b**ounds



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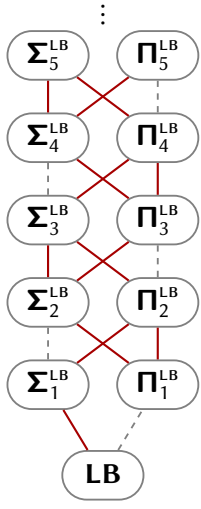

THEOREM: — Strict inclusions

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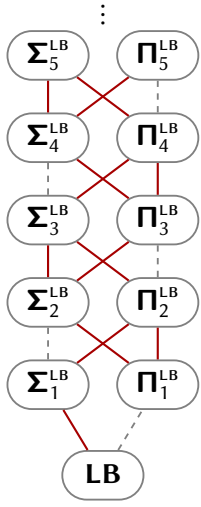

THEOREM: — Strict inclusions

--- Equalities

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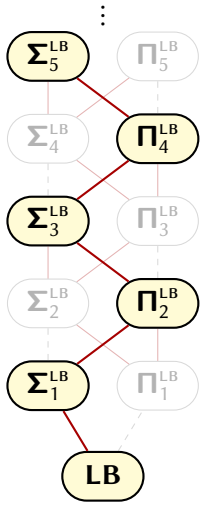

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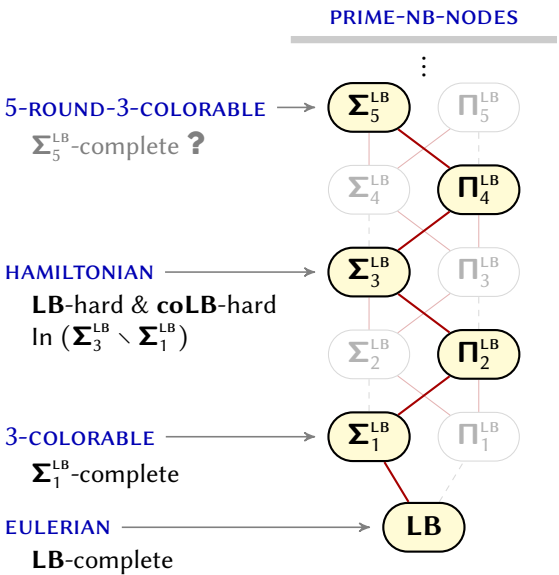

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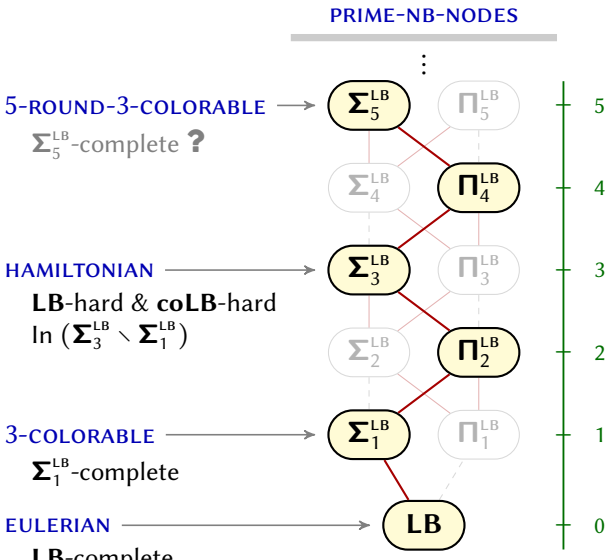

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A measure of locality?

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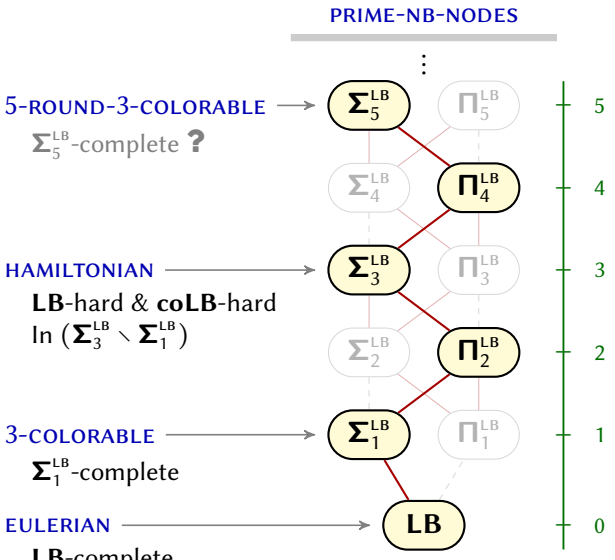

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THEOREM: — Strict inclusions

--- Equalities

We lose one thing:
Fagin's theorem

A measure of locality?

Building a theory

Image source: www.rhb.ch

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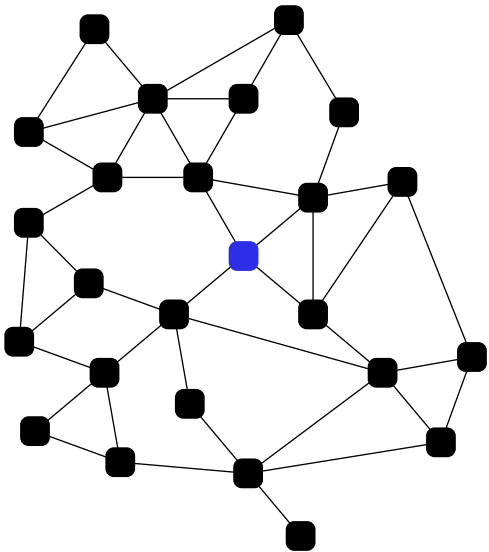
SOME-NODE-BLUE $\in \Sigma_3^{LB}$

To prove the existence of a **blue node**:



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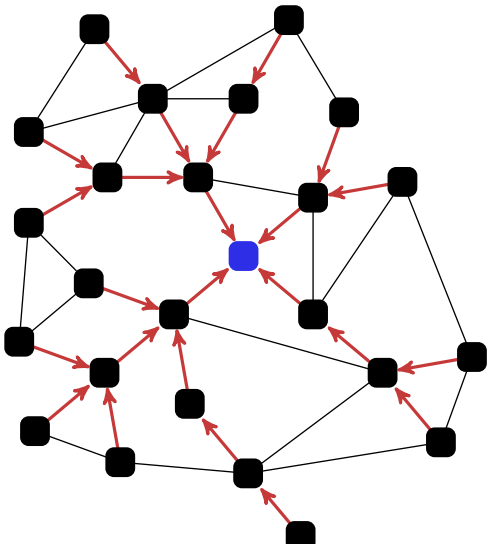

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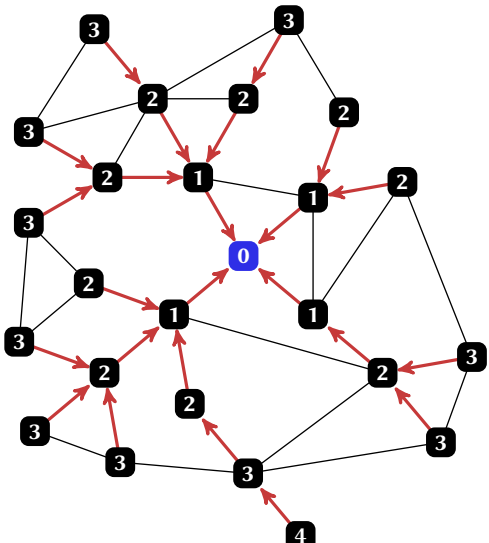

To prove the existence of a **blue node** 🔵:

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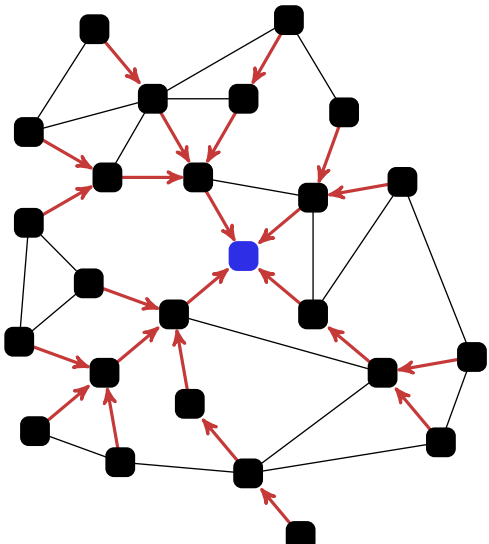

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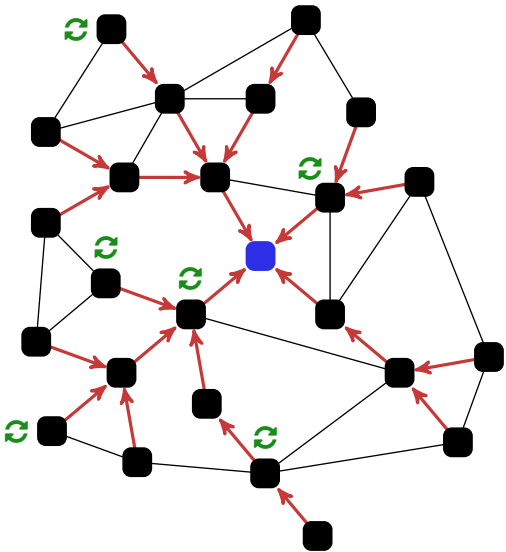

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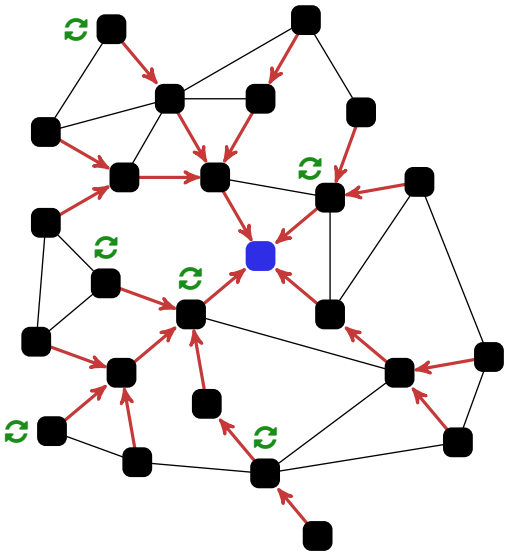

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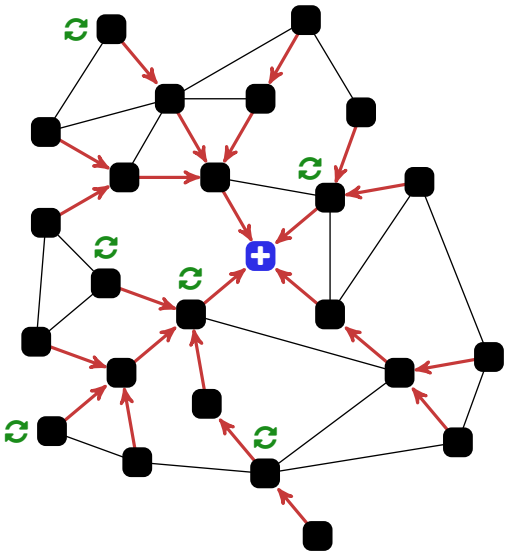

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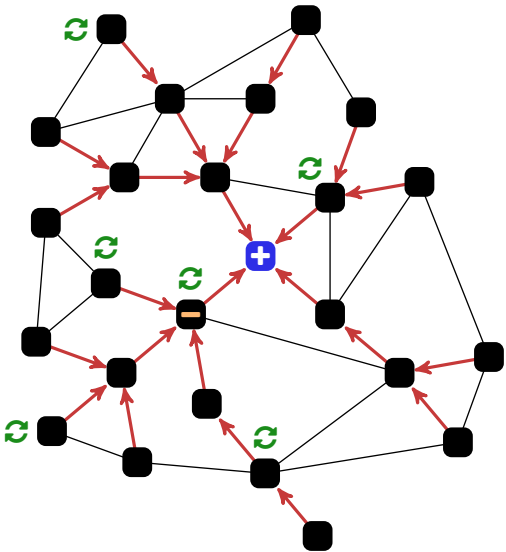

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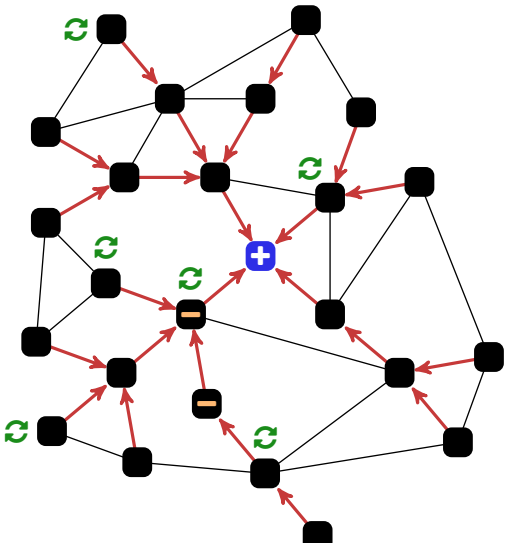

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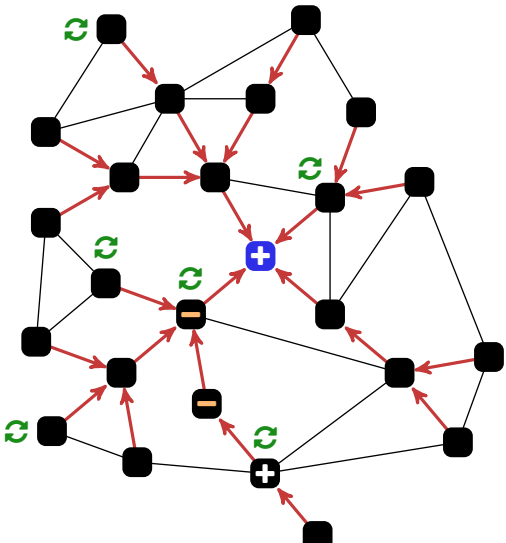

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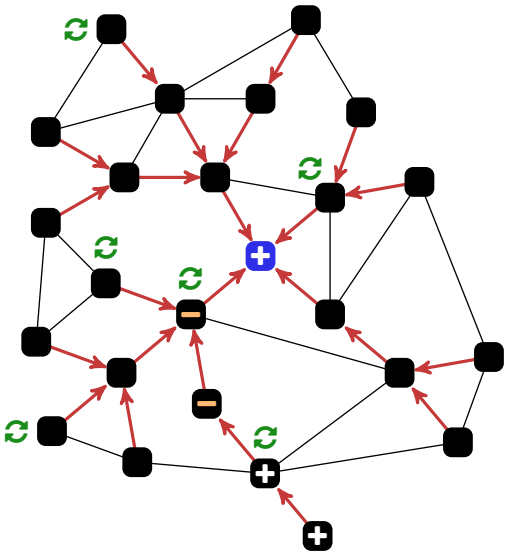

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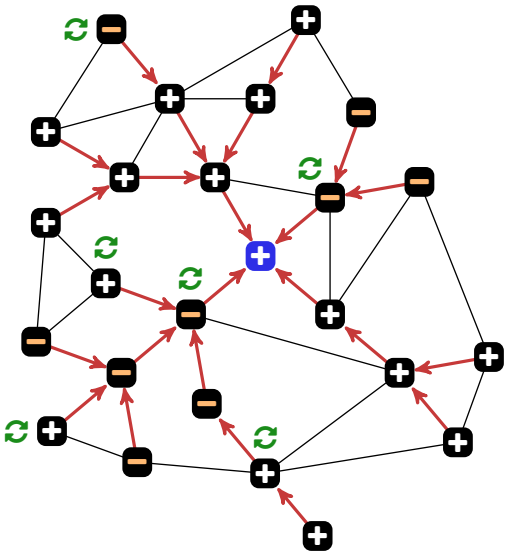

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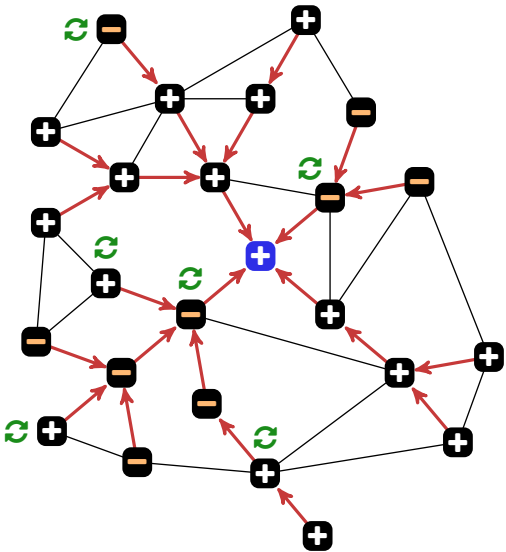
SOME-NODE-BLUE $\in \Sigma_3^{\mathrm{LB}}$



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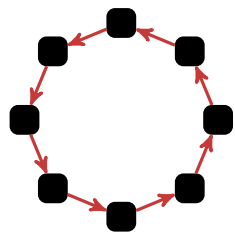

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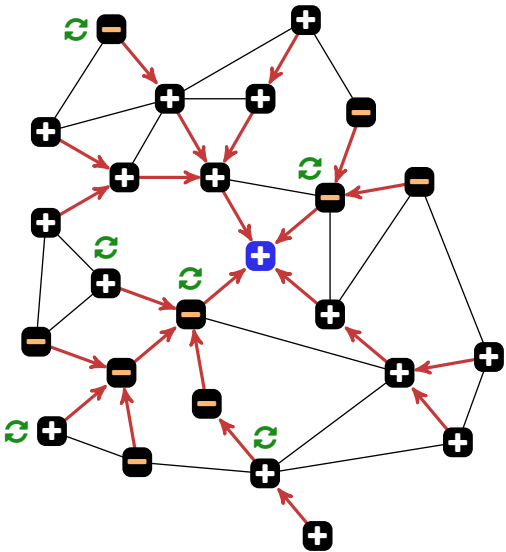

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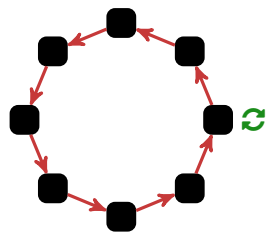

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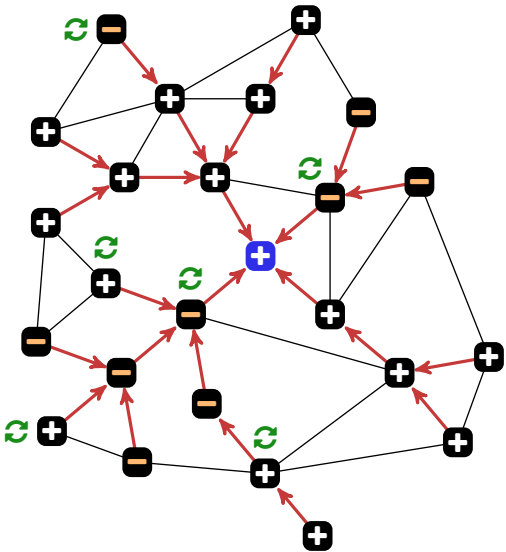

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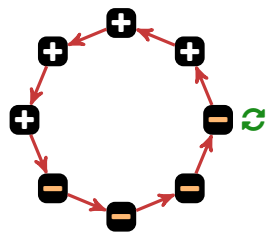

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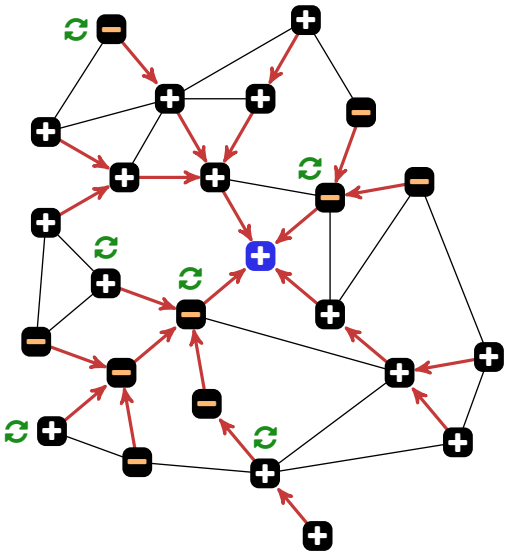

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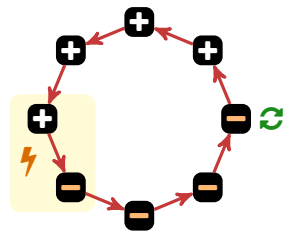
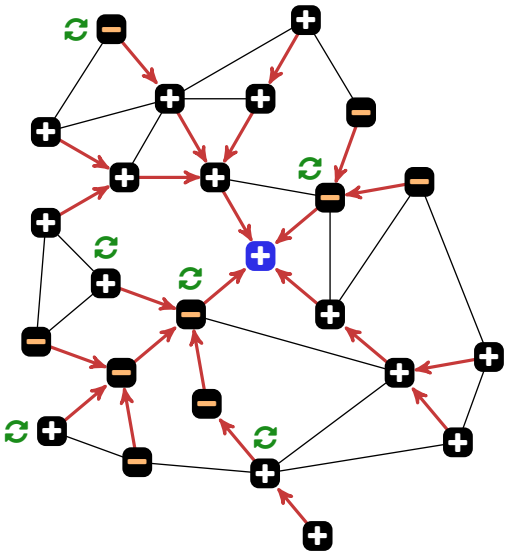


UNIQUE-BLUE-NODE $\in \Sigma_3^{LB}$



To prove that there is **exactly one** 🔵:

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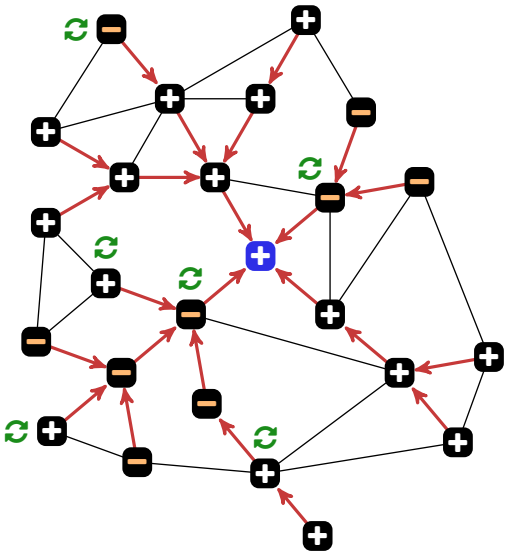
UNIQUE-BLUE-NODE $\in \Sigma_3^{LB}$



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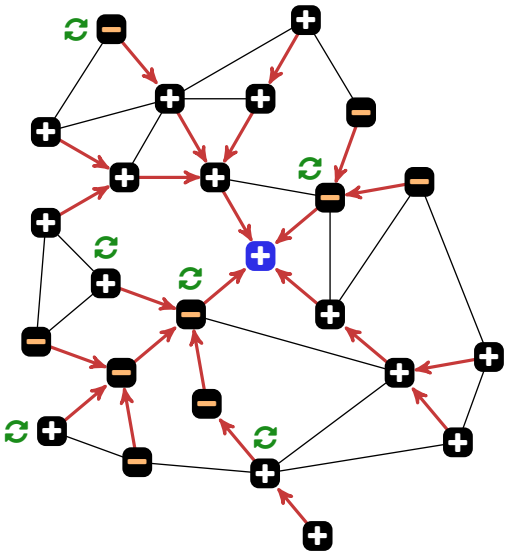
Eve tells each node if 🔵 is a flipping node.

UNIQUE-BLUE-NODE $\in \Sigma_3^{LB}$



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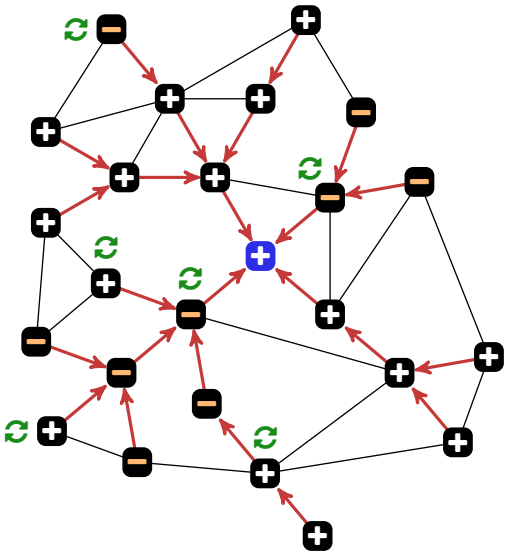
Eve tells each node if 🔵 is a flipping node.

UNIQUE-BLUE-NODE $\in \Sigma_3^{LB}$



To prove that there is **exactly one** 🔵:

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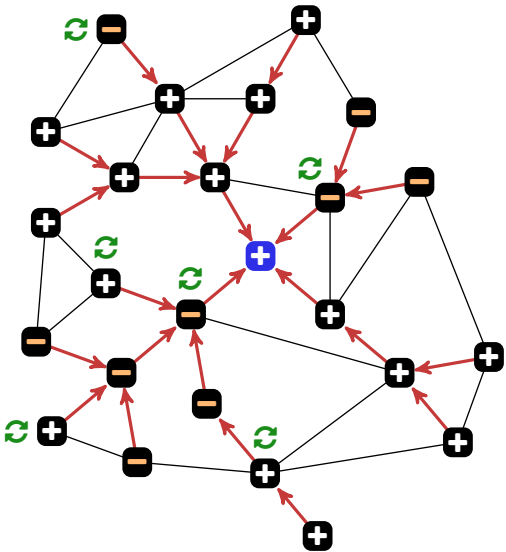

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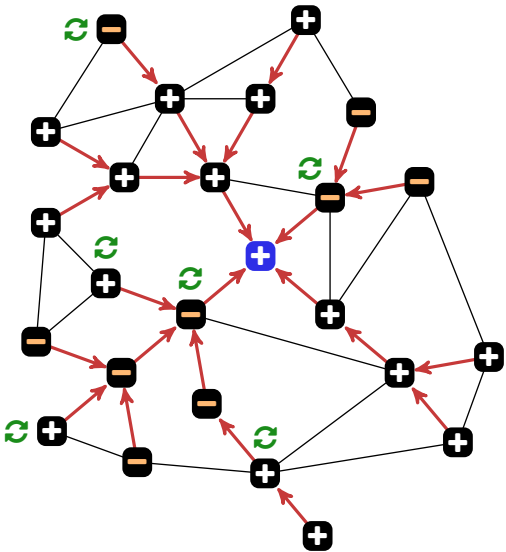

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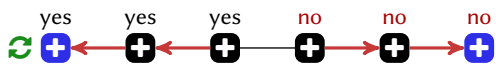

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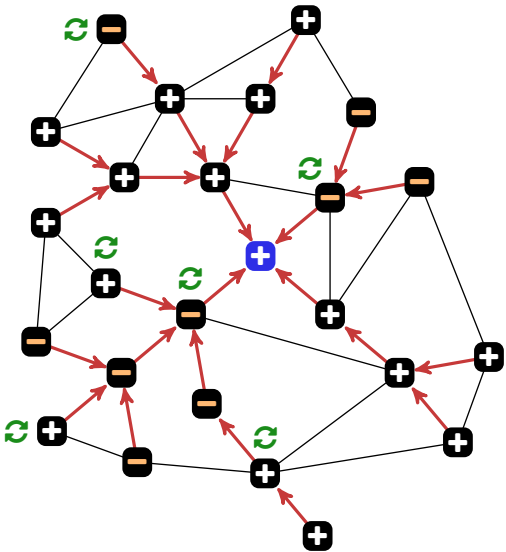

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Properties in Σ_3^{LB}

*With a **spanning tree**, Eve can prove many things:*

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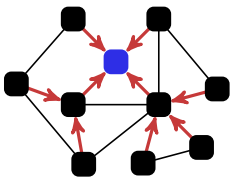

Examples adapted from Göös & Suomela (PODC 2011): *Locally Checkable Proofs*

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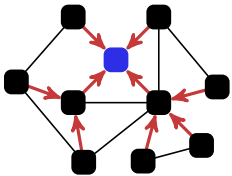
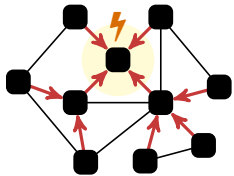


Any property in **coLB**



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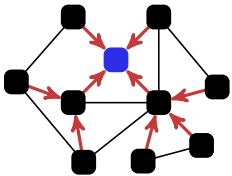

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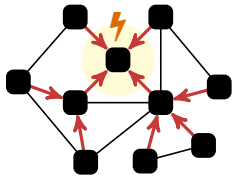
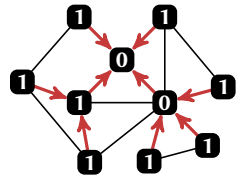


EVEN-NB-NODES



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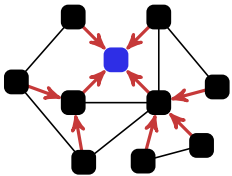

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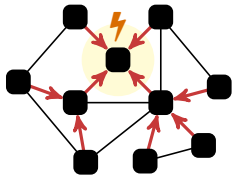

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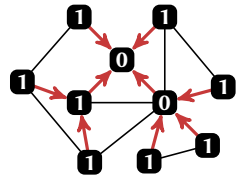
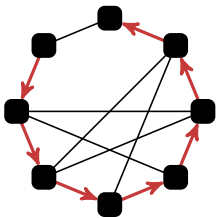


HAMILTONIAN



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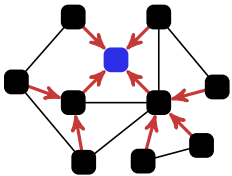

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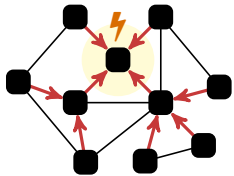

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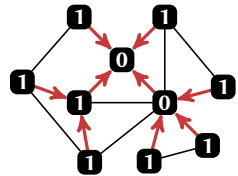

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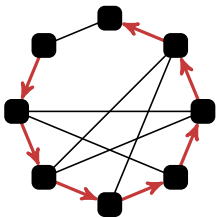
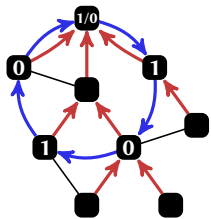


NON-2-COLORABLE



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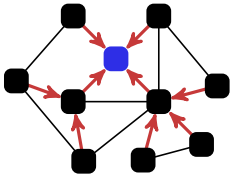

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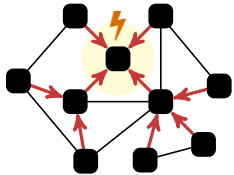

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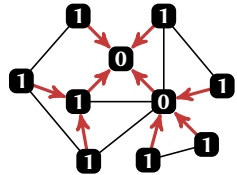

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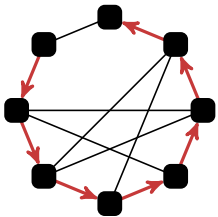

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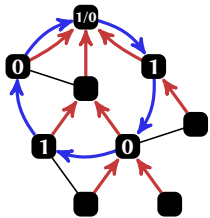
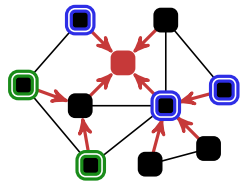


EMSO-definable properties



$\exists S_1, \ldots, S_k \; \exists x \; \forall y \; \varphi$

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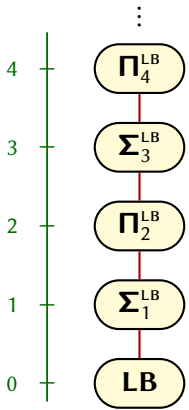
Measuring locality?



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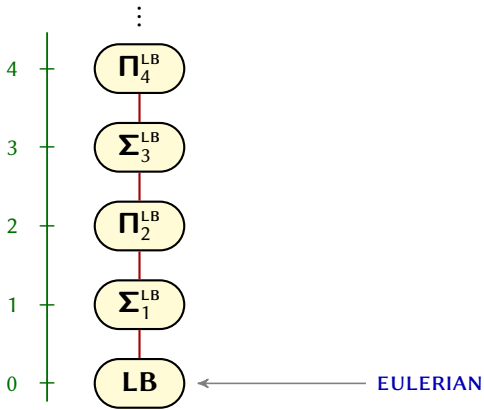

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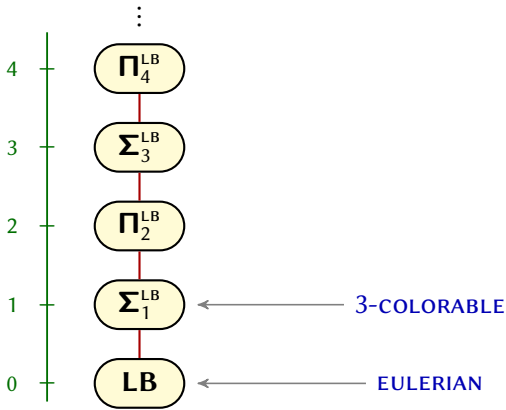

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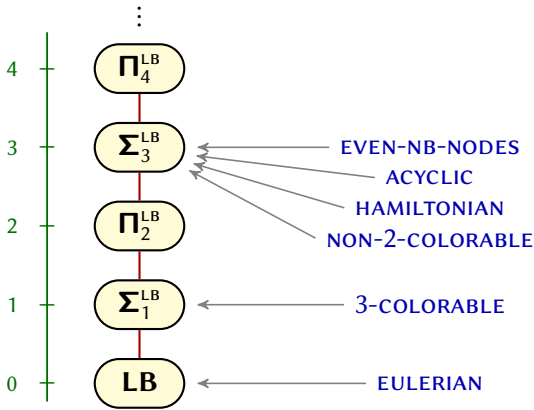

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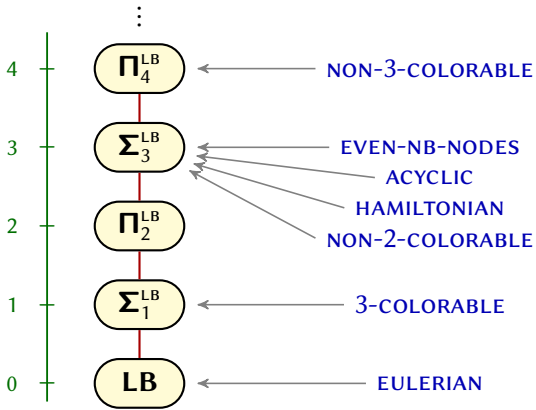

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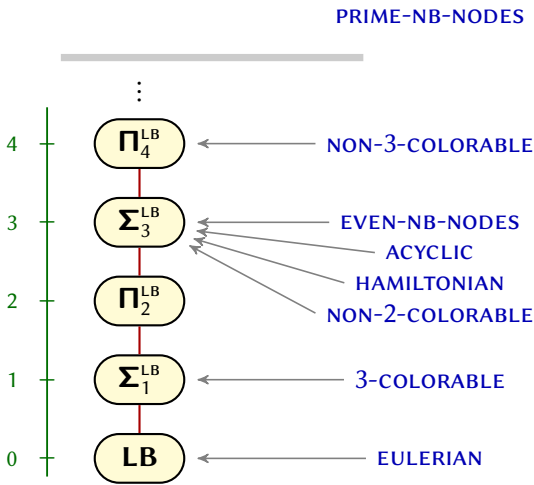

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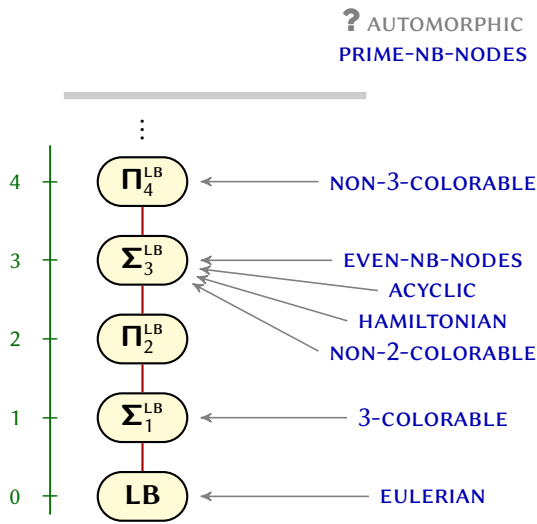

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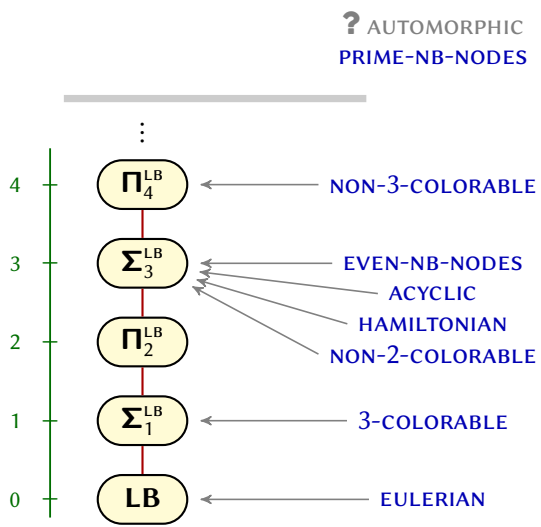

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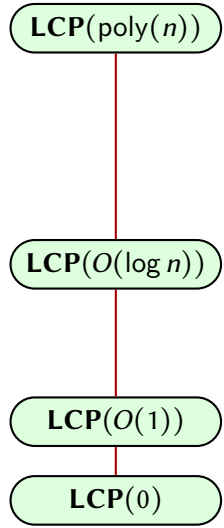

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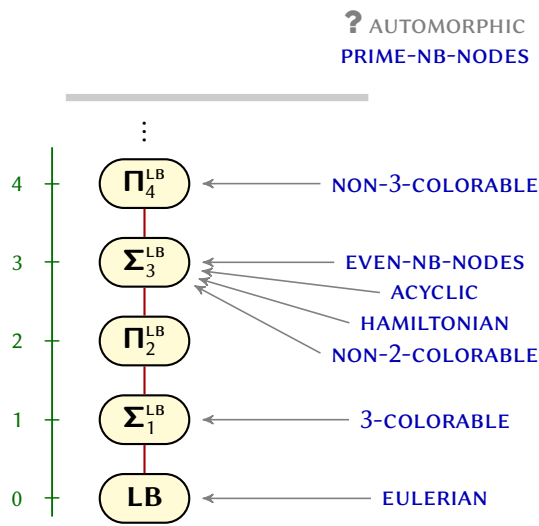

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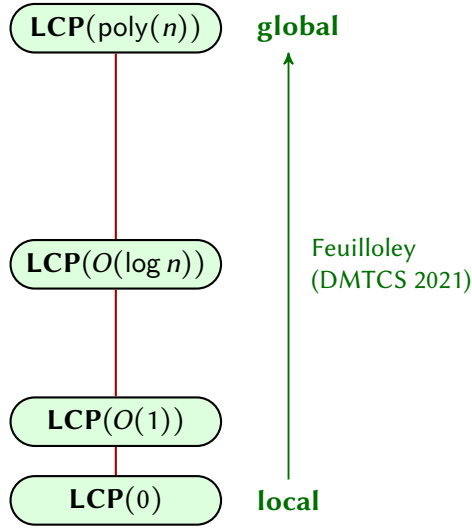

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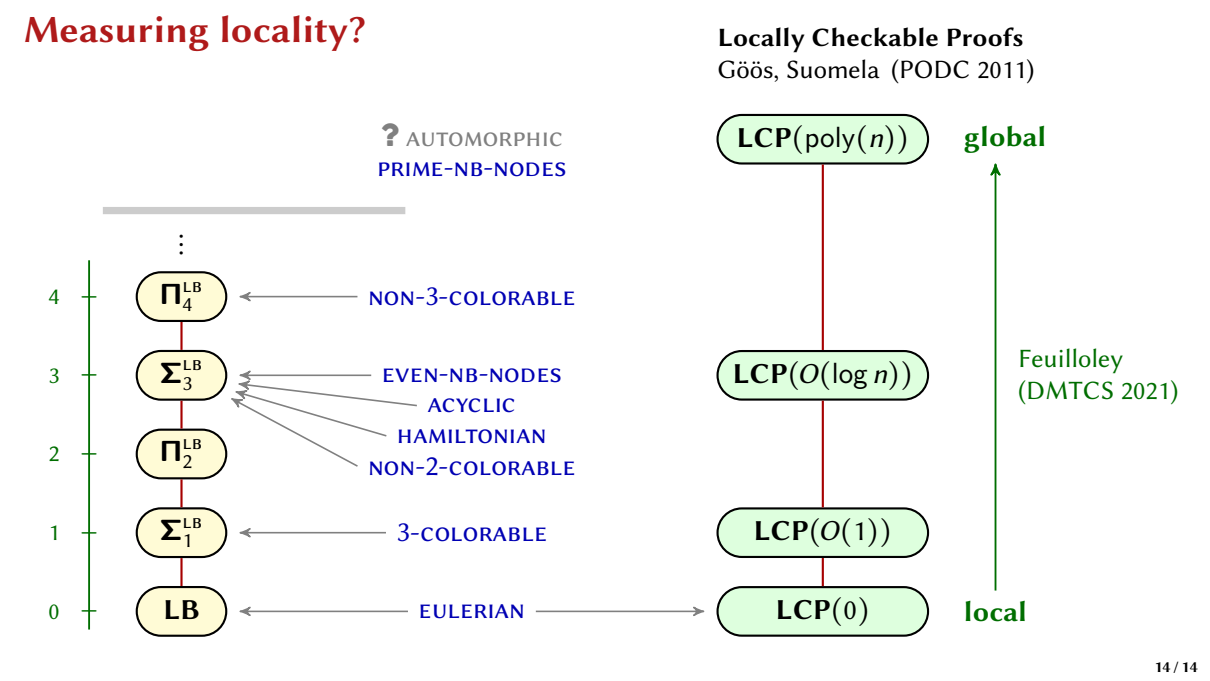

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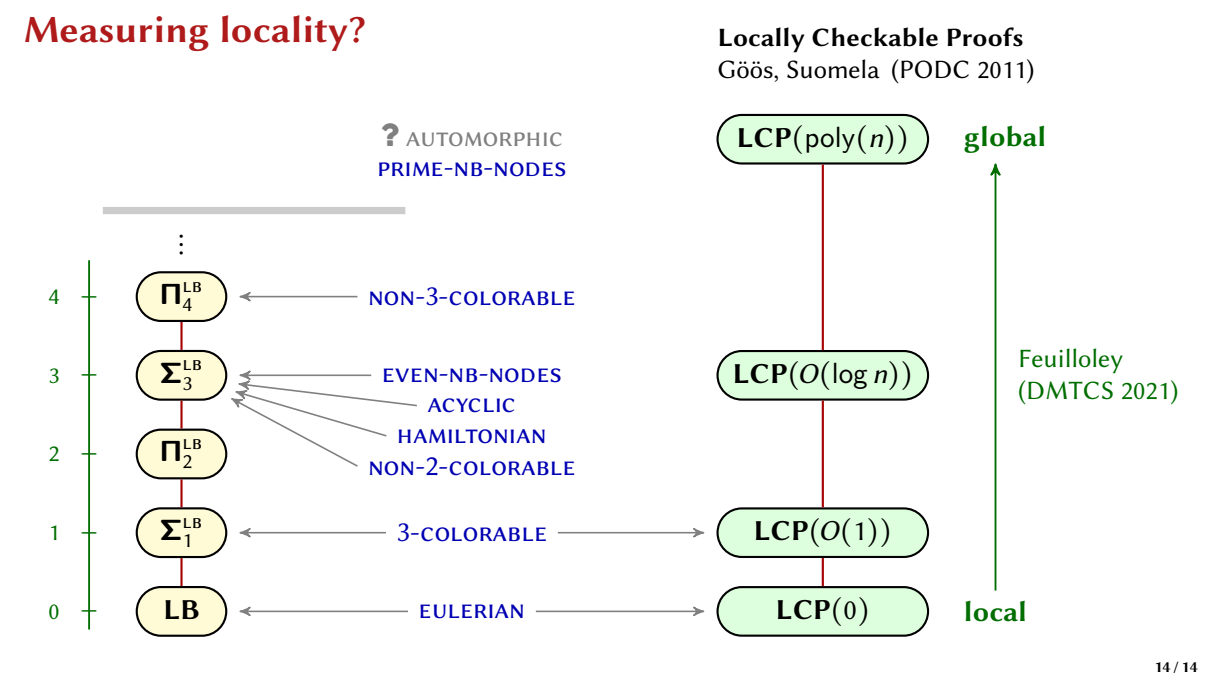

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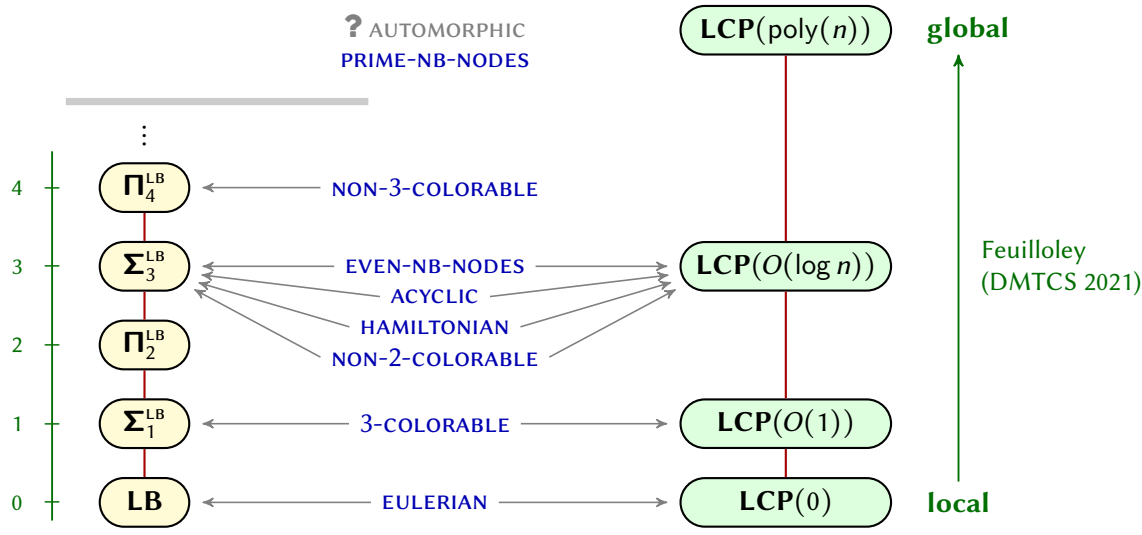

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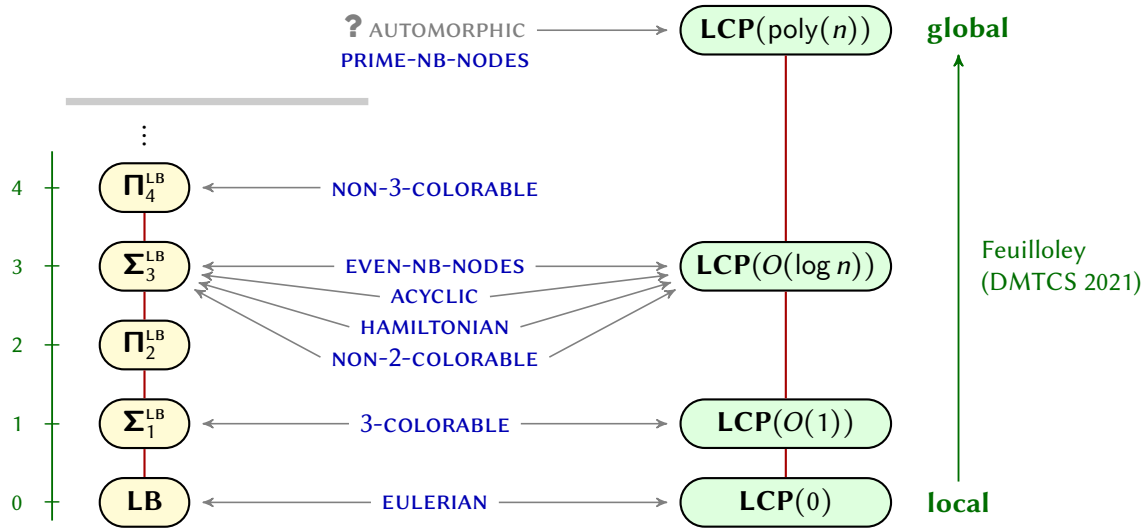

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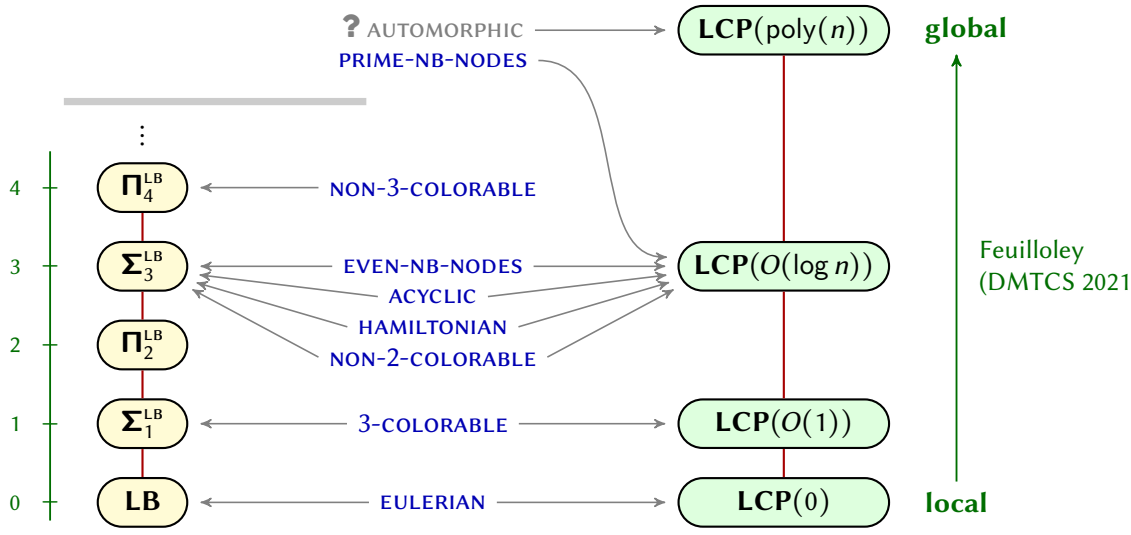

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