Aalto University
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Recent Breakthroughs in Distributed Quantum Computing

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ADGA 2025

Overview

Some basics

Separations

- Between quantum and randomized LOCAL
- Between quantum with and without pre-shared entanglement

Ruling out quantum advantage

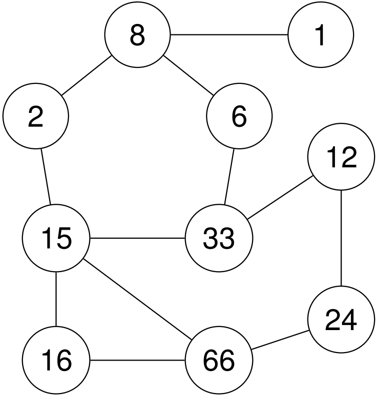
The frontier: 3-coloring cycles

Summary

Some basics

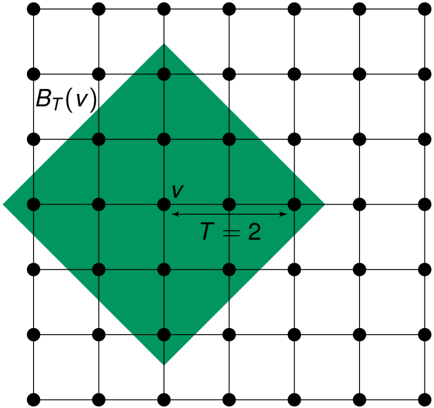
LOCAL model [Lin92]

- Every node is a computer (with unbounded computational resources)
- Every node has its own (unique) ID
- Communication graph $=$ input graph
- Synchronous, unbounded communication



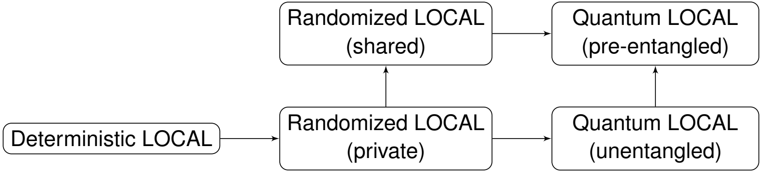
Complexity measure

- Measure: *number of communication rounds*
- If algorithm runs in T rounds, then essentially every node
  - gathers radius T ball $B_T(v)$ around it
  - and computes and outputs a function $f(B_T(v))$
- In other words, local computation is “for free”



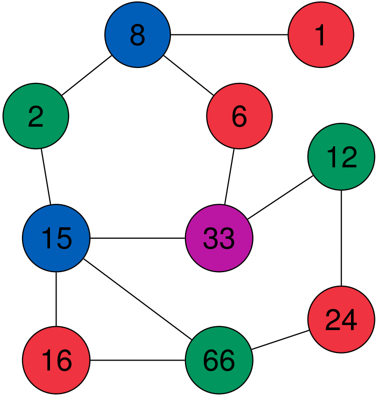
Direct extensions of LOCAL

- Nodes may have access to randomness
  - Private randomness: each node uses *its own* source
  - Shared randondmess: all nodes use *the same* source
- Nodes may be quantum computers and use quantum communication
  - Nodes may share an initial entangled state or not



What problems do we care about here?

- *Locally checkable labeling* (LCL) problems [NS95]
- Nodes output labels from some finite set Γ
- Must satisfy constraints (checkable in $O(1)$ radius)
- Examples: coloring, maxim**al** matching, maxim**al** independent set (MIS), sinkless orientation

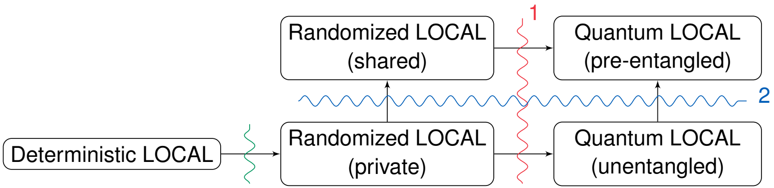


Separations

Full separation between extensions of LOCAL

Both were proven *this year*:

1. Randomized and quantum
2. Private/no entanglement and shared/pre-shared entanglement



- Separation between deterministic and randomized was already known (e.g., sinkless orientation)
- Separations all hold in *general graphs* (no extra assumptions!)

Separation 1: Quantum vs. randomized

STOC 2025 [Bal+25a]

There is an LCL problem that has the following complexities:

- $O(1)$ in quantum LOCAL (even without pre-shared entanglement)
- $\Omega(\Delta)$ in randomized LOCAL (even with shared randomness)

where Δ is the maximum degree of the input graph

SODA 2026 [Bal+25b]

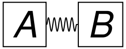
There is an LCL problem that has the following complexities:

- $O(\log n)$ in quantum LOCAL (even without pre-shared entanglement)
- $\omega(\log n)$ in randomized LOCAL (even with shared randomness)

Before these a separation was known but not for an LCL problem [LNR19]

Quantum games

Famous game in quantum literature: *Clauser-Horne-Shimony-Holt (CHSH) game*



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$\boxed{A}$ $\boxed{B}$

Quantum games

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- $x, y \in \{0, 1\}$

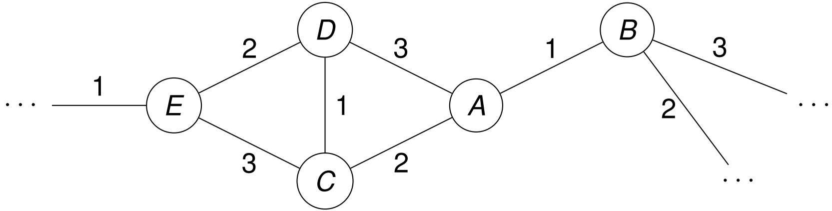
Quantum games

Famous game in quantum literature: *Clauser-Horne-Shimony-Holt (CHSH) game*

- $x, y, a, b \in \{0, 1\}$
- Winning condition: $a \oplus b = x \wedge y$
- It is a famous result that there is a quantum strategy that wins with $\approx 85\%$ probability whereas classically the best is 75% (assuming inputs are uniformly random)

Network of quantum games

- Hard game (CHSH) $\rightsquigarrow$ hard distributed problem



- Δ-regular graph with Δ-edge-coloring given

Network of quantum games

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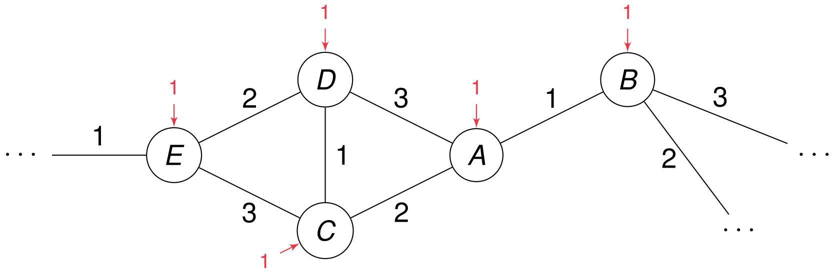

- Δ-regular graph with Δ-edge-coloring given
- Every node X receives input 1

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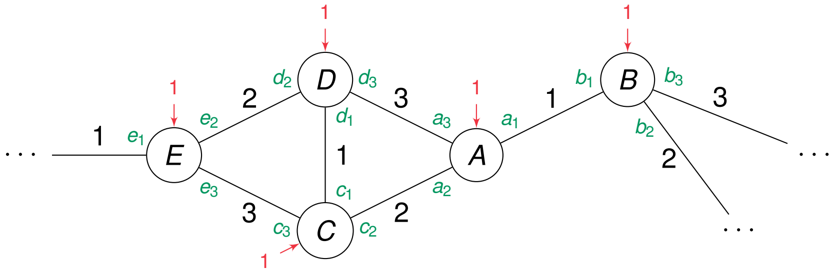
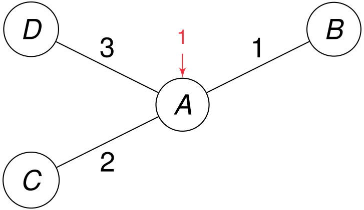


- Δ-regular graph with Δ-edge-coloring given
- Every node X receives input 1 and produces Δ outputs $x_1, \ldots, x_\Delta$
- Constraint at each edge:
  - If edge connects X and Y and is labeled $i \in [\Delta]$, then $x_i \oplus y_i = x_{i-1} \wedge y_{i-1}$
  - We set $x_0 = 1$ (input)

Network of quantum games: complexity

- This problem can be solved in $O(1)$ rounds in quantum



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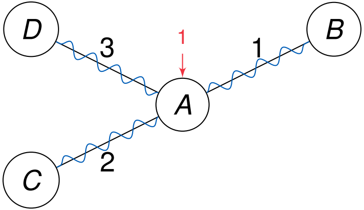

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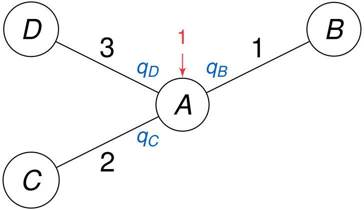

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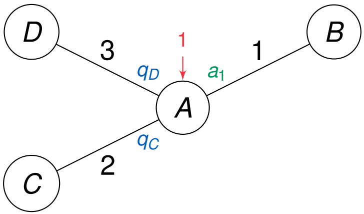

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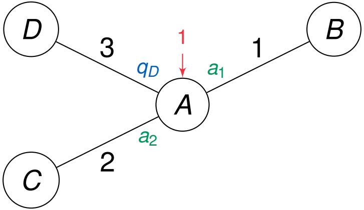

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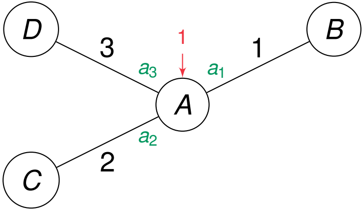

- We can prove: randomized LOCAL (even with shared randomness) needs $\Omega(\Delta)$ rounds
  - Complex proof based on round elimination
  - Open: Is there a simpler hardness argument?

However, there is a caveat...

Getting an actual separation

- Big caveat: quantum only succeeds in solving CHSH with $\approx$ 85% probability
- Probability all edges around a single node are correct: $\approx 0.85^{\Delta}$
- Probability all edges are correct: $\approx 0.85^{\Omega(\Delta n)} \overset{n\to\infty}{\to} 0$ ☹

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- The catch is that GHZ is not a two-party but *three-party* game
- Hence we need to move to 3-hypergraphs
- Round elimination proof also gets much more complicated 😕

Details: [Bal+25a]

From a separation on Δ to a separation on n [Bal+25b]

- Core of the problem remains the same, complexity is translated from Δ to n
  - Technique could be useful in other contexts (even unrelated to quantum)

- Two key ideas:
  1. Every node is turned into a tree of height $\Theta(\log n)$ (plus extra edges to help with certification)
  2. Every one of its Δ edges is replaced by a gadget with diameter $\Theta(\log n)$

- Quantum works in parallel and needs only $\Theta(\log n) + \Theta(\log n) = \Theta(\log n)$ rounds
- Classical must use sequential strategy and needs $\Omega(\Delta \log n)$ rounds, which is $\omega(\log n)$ for an appropriate choice of Δ

Separation 2: Private vs. shared

ICALP 2025 [Bal+25d]

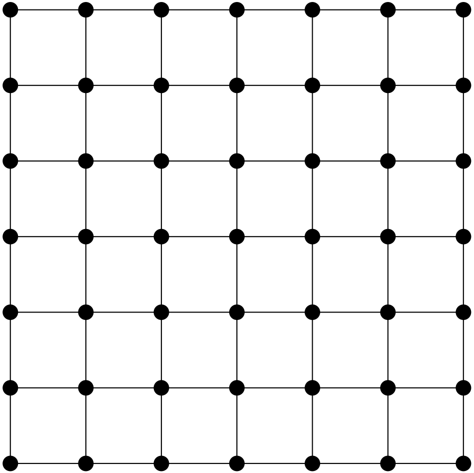
There is an LCL problem that has the following complexities:

- $O(\log n)$ in randomized LOCAL with shared randomness
- $\Omega(\sqrt{n})$ in quantum LOCAL (without pre-shared entanglement)

(assuming nodes are given the total number n of nodes in the graph)

- Basic idea is to define a problem where global coordination is needed (nodes need to globally agree on a solution)

The main idea



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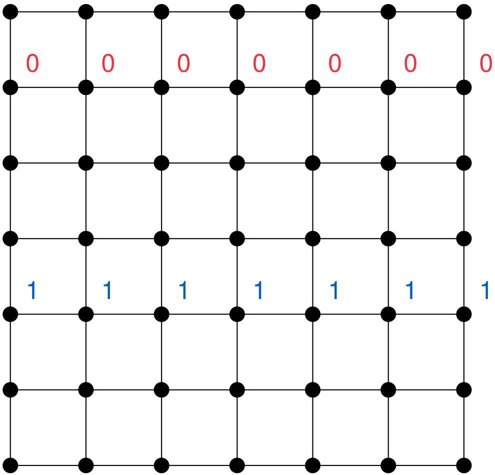

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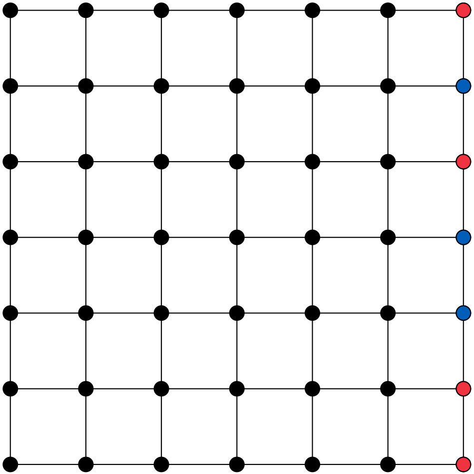

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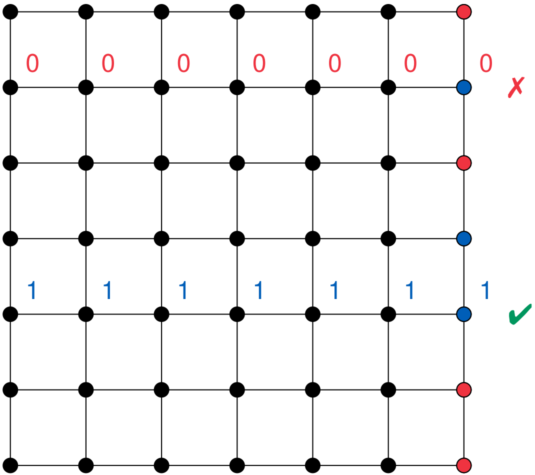

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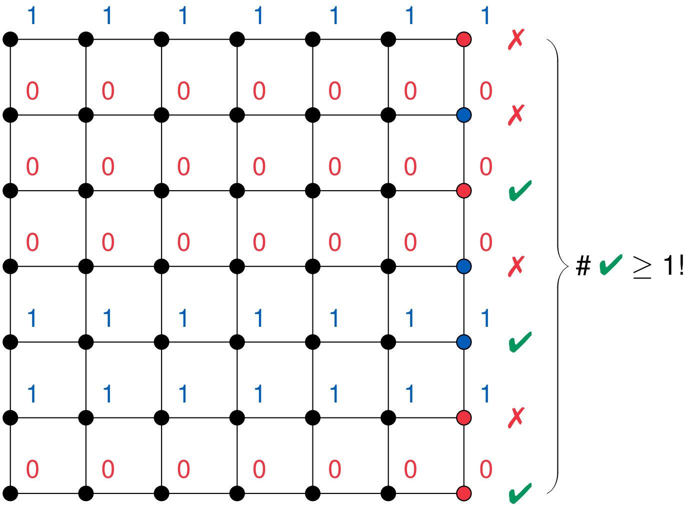

How to get a separation

- If we have shared randomness (1 bit per row is enough): every node in the same row outputs the same bit $b \in \{0, 1\}$
  - If the grid is tall enough, say $\Omega(\log n)$ height, then failure probability is $1 - 2^{-\Omega(\log n)} = 1/n^{\Omega(1)}$
  - Use $O(\log n)$ rounds to check grid is large enough

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- If we only have private randomness, then:
  - Outputting a *fixed* bit $b \in \{0, 1\}$ is definitely bad
  - Coordinating independent choices along a row (even with quantum) is not possible
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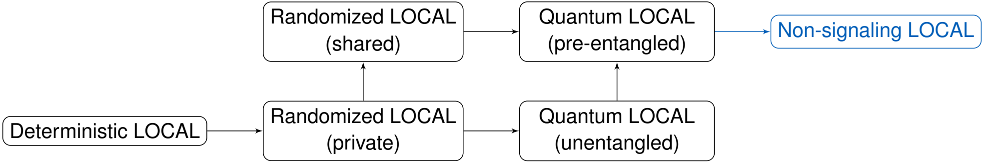
- Hard part: We expect some structure for upper bound to work, but we want a separation on *general graphs*
  - E.g., we *do not* have the promise that we are on a grid
  - Relies on LCL certification techniques from previous works

Details: [Bal+25d]

Ruling out quantum advantage

Ruling out quantum advantage

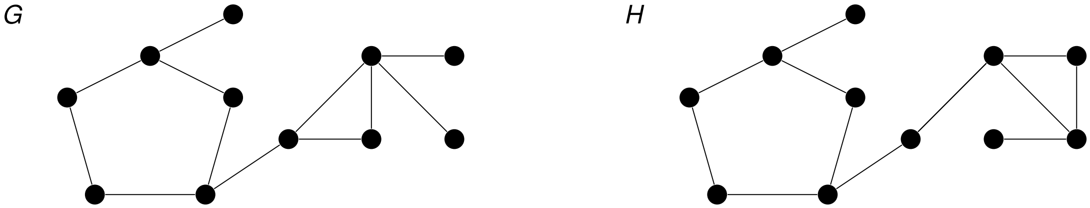
- Currently, lower bounds for quantum are actually lower bounds for a *stronger model*



- Advantages:
  - Lower bounds for non-signaling can be based on graph theory (e.g., propagation arguments)
  - Non-signaling is a *stronger* model, so we also get *stronger* lower bounds
- Disadvantages:
  - Lower bounds for non-signaling may not directly tell us much about quantum
  - There are (very prominent!) problems where there is a trivial upper bound in non-signaling (coming up later)

Non-signaling LOCAL [AF14; GKM09]

- Non-signaling defined not on terms of explicit model but of *property* of a distribution:
  - A problem can be solved in non-signaling LOCAL with locality T if there is a (*global*) random process $\mathcal{A}$ that is non-signaling with locality T that solves it (whp)
- Random process $\mathcal{A}$ is non-signaling with locality T if the following holds:



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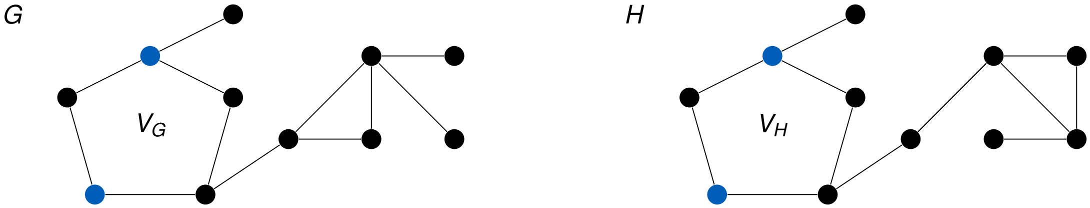

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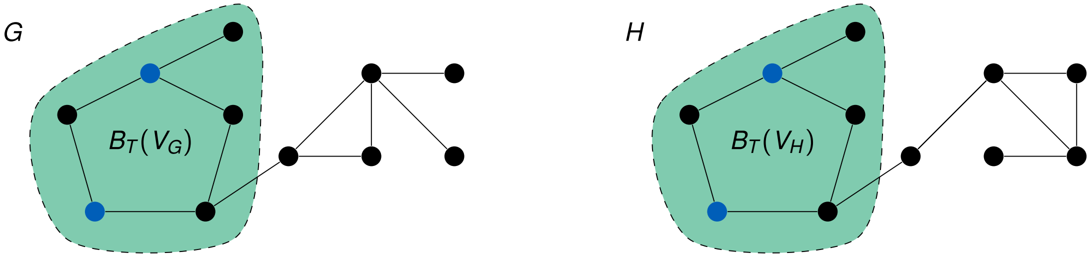

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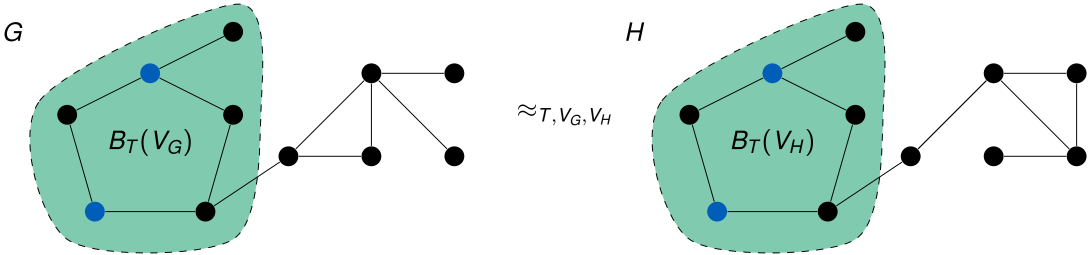


...then marginals $\mathcal{A}(G)|_{V_G}$ and $\mathcal{A}(H)|_{V_H}$ are *identically distributed*

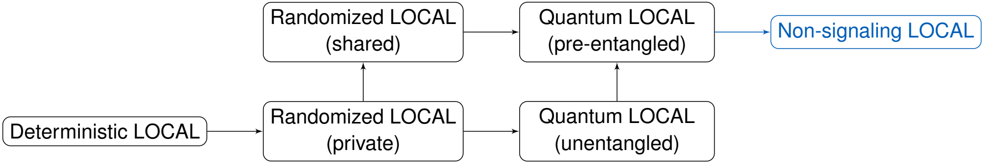
Some prominent results

For the following problems, the locality of *deterministic* and non-signaling LOCAL are the same (up to polylog factors):

STOC 2024 [Coi+24]: approx coloring, i.e., c-coloring a χ-chromatic graph (where $c \geq \chi$)

DISC 2025 [Bal+25c]: linear programming

- Hence also approx vertex cover or approx maximum matching

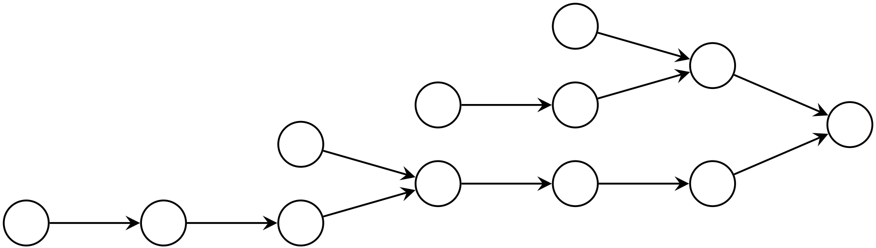


Lower bounds beyond non-signaling

There are also recent results that hold not only in non-signaling (and hence also in quantum) LOCAL but in even stronger, *centralized* models, e.g.:

STOC 2025 [Akb+25]

The following holds in *rooted trees*: If Π is solvable with $o(\log\log\log n)$ locality in *randomized online-LOCAL*, then it is solvable with $O(\log^* n)$ locality in *deterministic LOCAL*.



The frontier: 3-coloring cycles

Symmetry-breaking problems

- $\Theta(\log^* n)$ is class of *symmetry-breaking* problems in deterministic / randomized LOCAL
  - Classical representatives: 3-coloring a cycle, maximal independent set (MIS)
- "Meta-solution" to problems in this class [Bra+17]:
  1. **Break symmetry** — find distance-k coloring (i.e., coloring of G^k) for a certain $k = O(1)$
  2. **Solve problem** — use coloring as "anchor" to place labels
- E.g., we can solve MIS by choosing all nodes with color 1, then augment by greedily adding nodes colored 2, etc.
  - If we have MIS, we can 3-color a cycle: Place 3's on MIS nodes and complete in-between with a 2-coloring
- Only the first item takes $\Theta(\log^* n)$, the second one is constant-time!

- Question: Is complexity still $\Theta(\log^* n)$ in quantum or stronger models?

Symmetry-breaking for free

STOC 2025 [Akb+25]

If Π is solvable with $O(\log^* n)$ locality in *deterministic LOCAL*, then it is solvable with $O(1)$ locality in the *bounded-dependence* model.

- Bounded-dependence is a weaker version of non-signaling
  - Also not an explicit model but defined in terms of random processes
  - Bounded-dependence for $O(1)$ locality is also known as *finitely dependent*

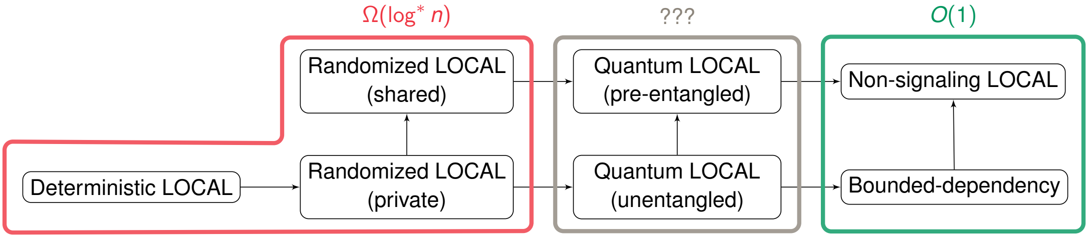
Corollary

If Π is solvable with $O(\log^* n)$ locality in *deterministic LOCAL*, then it is solvable with $O(1)$ locality in non-signaling LOCAL.

- Hence 3-coloring cycles in non-signaling is “trivial”

The frontier: 3-coloring on cycles

- Summarizing what we know about symmetry-breaking problems:



- To settle the middle we need *truly* quantum techniques (!!)
- Just showing a lower bound in bounded dependence or non-signaling is not possible

Summary

Summary

1. *Full* separations between all “direct” extensions of LOCAL
    - Separations are on *general graphs*
    - Possibly room for improvement, e.g.:
        - Gap between quantum and randomized is very small
        - Can we simplify the lower bound proofs?
        - Do these separations hold for *natural* problems?

2. Strong results showing no quantum advantage for *interesting* problems
    - Approximate coloring and linear programming
    - Results apply to non-signaling, so *very strong* lower bounds

3. Wide open: 3-coloring cycles
    - *No lower bound based on non-signaling possible*
    - Techniques?

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