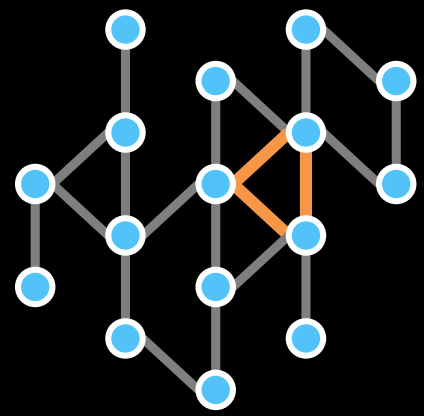
Distributed Subgraph Finding

Keren Censor-Hillel, Technion

ADGA 2025

Distributed subgraph finding

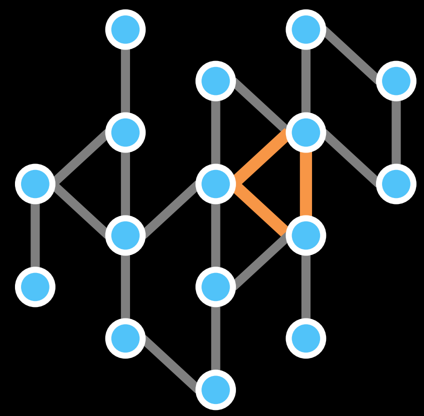
- Structures in social networks
- Processing large graphs



Distributed subgraph finding

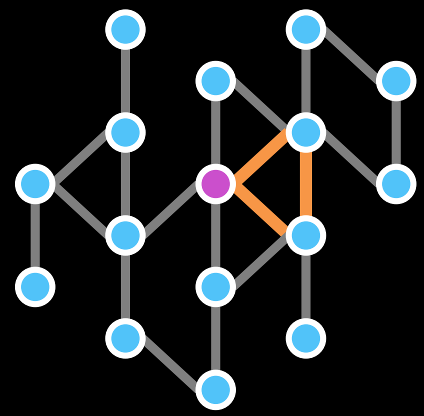
- Structures in social networks
- Processing large graphs

  - Coloring triangle-free graphs
    [Pettie, Su ‘13]
  - Large cuts in triangle-free graphs
    [Hirvonen, Rybicki, Schmid, Suomela ‘17]



Locality

- Each **node** knows only its neighbors
- Synchronous commmunication rounds



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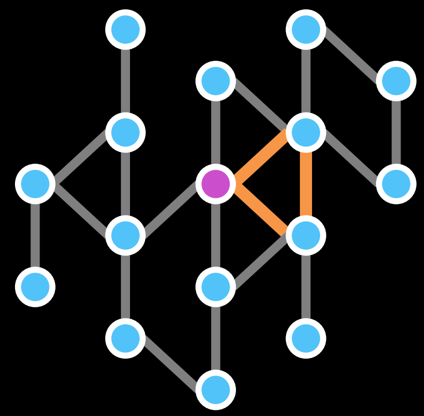
- Synchronous commmunication rounds

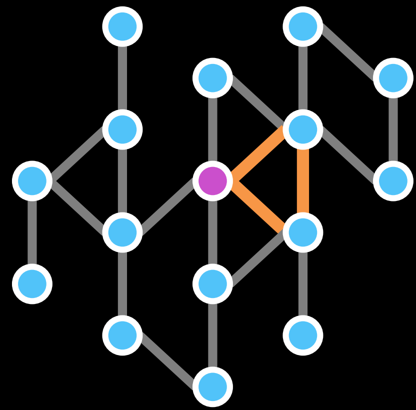
$O(p)$ rounds for finding
any subgraph H of diameter p

The big question: limited bandwidth?



Bandwidth

Message size:
- Local: Unbounded [Linial '92]
- Congest: $O(\log n)$ bits [Peleg '00]



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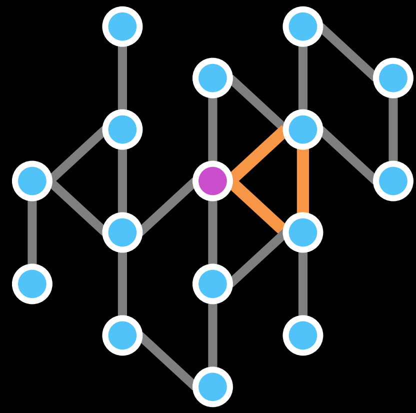

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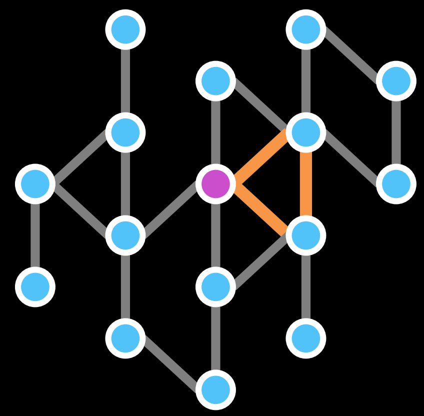

This talk: Triangles in congested models
+ Glimpse into the broader context

Listing Triangles

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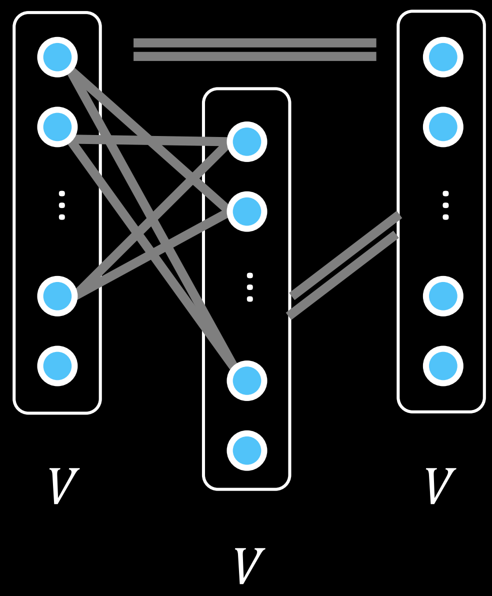

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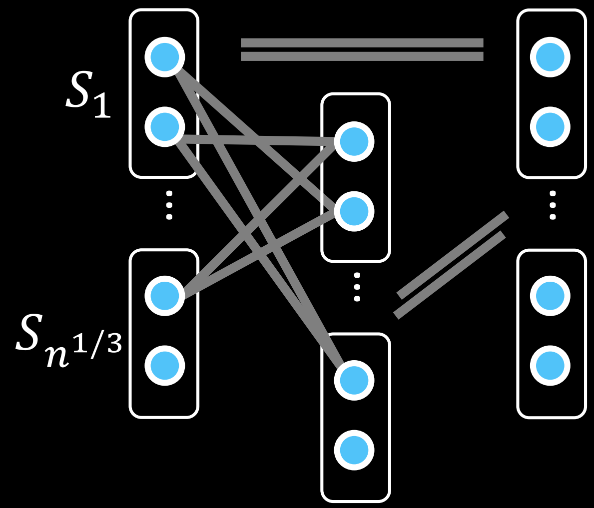

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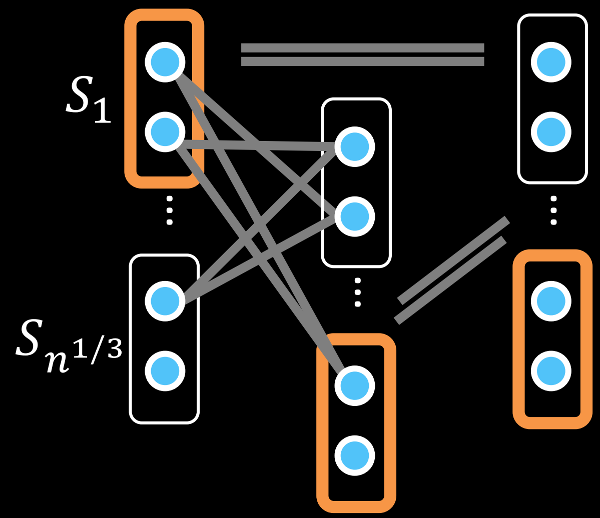

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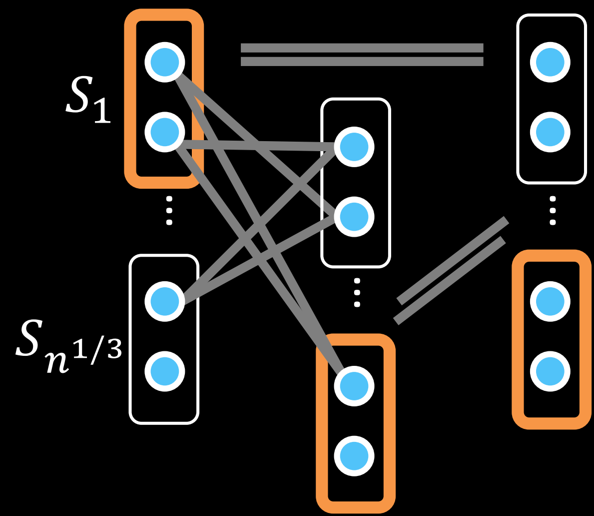

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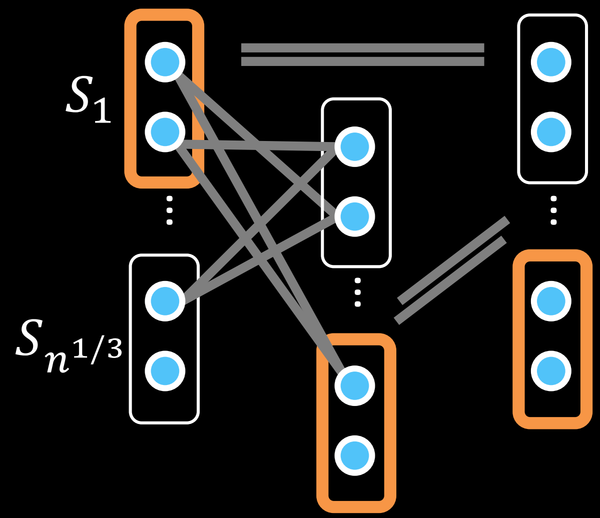
$O(n^{1/3})$ rounds



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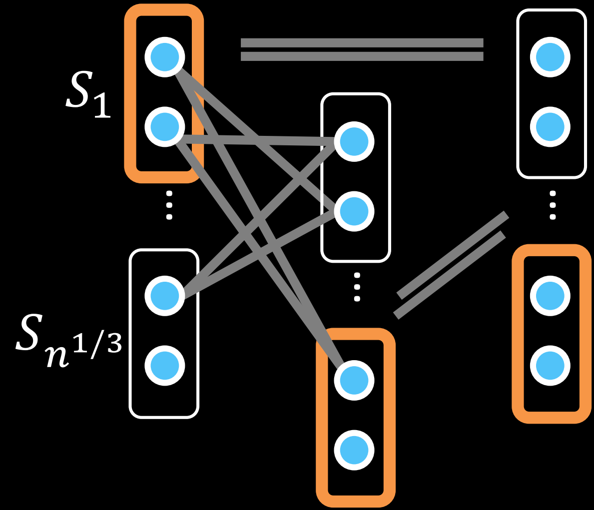

- Routing scheme: send/receive n messages per node in O(1) rounds [Lenzen '13]

Listing Triangles

Triangle listing: every triangle is output by some node

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- $\tilde{\Omega}(n^{1/3})$ rounds
  [Izumi, Le Gall '17, Pandurangan, Scquizzato, Robinson '18]
  - $\Omega(n^{4/3})$ for the entropy of the transcript a node sees in $G_{n,1/2}$
  - A node can receive at most $\tilde{O}(n)$ bits per round

Listing Triangles in Sparse Graphs

Triangle listing in sparse graphs:

- $O(m^2/n^3)$ rounds (m = edges)

[Dolev, Lenzen, Peled '12]

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- $\widetilde{O}(m/n^{5/3})$ rounds:
  DLP with a random partition and random load balancing of who gets what
- $\widetilde{\Omega}(m/n^{5/3})$ rounds

[Pandurangan, Scquizzato, Robinson '18]

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[Pandurangan, Scquizzato, Robinson '18]

- $O(m/n^{5/3})$ rounds, deterministic:
  sparse matrix multiplication in $O((\rho_S \rho_T/n)^{1/3})$ rounds
  (ρ_A= average number of non-zero elements per row)

[C, Leitersdorf, Turner '18]

Listing Triangles with Bounded Memory

[Ben Basat, C, Chang, Han, Leitersdorf, Schwartzman '25]

$\mu =$ bits of memory per node

such that $n \leq \mu \leq n^{4/3}$

- Triangle listing in $n^{1+o(1)}/\mu^{1/2}$ rounds

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With linear memory $\mu = n$ we need $n^{1/2}$ rounds

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- Corresponding lower bound

Detecting Triangles

Triangle detection: $\exists$ triangle $\Leftrightarrow$ $\exists$ node that outputs yes

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- $O(n^{0.158})$ rounds, ring matrix multiplication
  $O(n^{1-2/\omega})$, ω = matrix multiplication exponent
  $O(n^{\rho})$, ρ = matrix multiplication exponent in congested clique
  [C, Kaski, Korhonen, Lenzen, Paz, Suomela '15]

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- Super-constant lower bounds imply breakthroughs in circuit complexity
  [Drucker, Kuhn, Oshman ‘14]

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Open problem:
Round complexity of **triangle detection** in Congested Clique?

Detecting with T Triangles - Sampling

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$$\left.\boxed{\begin{array}{l}\text{If guess is good: prob. for specific triangle is } 1/(nT) \\ n \text{ experiments per guess, } T \text{ triangles to find}\end{array}}\right\} \text{correctness}$$

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correctness

Each subgraph has $n^{4/3}/T^{2/3}$ edges

Rounds: $\tilde{O}(n^{1/3}/T^{2/3})$

complexity

- Routing scheme: send/receive n messages per node in O(1) rounds [Lenzen '13]

Detecting with T Triangles: Faster

$\tilde{O}(\min\{n^{\rho}x^{-2+\delta(\rho-1)}, n^{0.1567}x^{-0.4617}\})$ rounds [C, Even, Vassilevska-Williams '24]

Refined parameters: V_T = nodes in triangles, $x = |V_T|$
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- Fast Matrix Multiplication on induced subgraph

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➔Repeat $\sim x^{\delta}$ times

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Mutliple FMM in parallel: Extend techniques of
[C, Kaski, Korhonen, Lenzen, Paz, Suomela '15] and [Le Gall '16]

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- Sample sets of vertices with probability $1/x$
- Rectangular Matrix Multiplication: find triangle that intersects sample

Detecting with T Triangles: Faster

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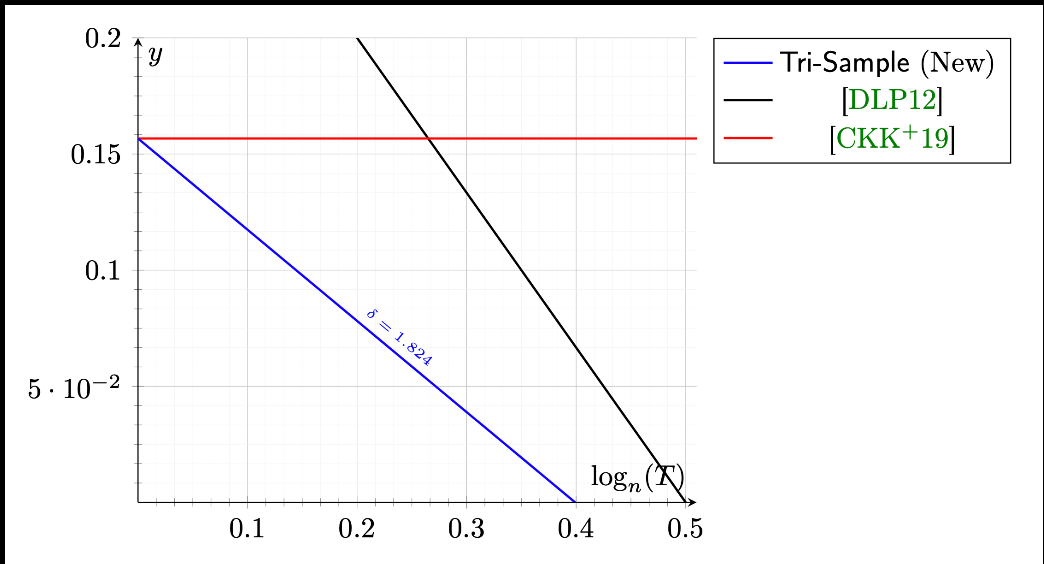

$\tilde{O}(n^{0.1567}/(t^{0.3926}+1))$ rounds in the worst case

➔ $\tilde{O}(1)$ rounds already when $t \geq n^{0.3992}$

*Slide courtesy of Tomer Even

Matrix Multiplication in Congested Clique

$O(n^{0.373})$	randomized	[Drucker, Kuhn, Oshman '14]
$O(n^{0.158})$	ring	[C, Kaski, Korhonen, Lenzen, Paz, Suomela '15]
$O(n^{1/3})$		
$\tilde{\Omega}(n)$	BCC	
$O((\rho_S \rho_T / n)^{1/3})$		[C, Leitersdorf, Turner '18]
$O((\rho_S \rho_T \rho_{ST})^{1/3} / n^{2/3})$		[C, Dory, Korhonen, Leitersdorf '19]
$O((\rho_S \rho_T \rho)^{1/3} / n^{2/3} + \log n)$		
Rectangular, multiple instances, more		[Le Gall '16]

Triangles in Congest

$\tilde{\Omega}(n^{1/3})$	listing	[Izumi, Le Gall '17, Pandurangan, Scquizzato, Robinson '18]
$\tilde{O}(n^{2/3})$	detection	[Izumi, Le Gall '17]
$\tilde{O}(n^{3/4})$	listing	
$\tilde{O}(n^{1/2})$	listing	[Chang, Pettie, Zhang '19]
$\tilde{O}(n^{1/3})$	listing	[Chang, Saranurak '19]
$\tilde{O}(n^{2/3})$	listing, det.	[Chang, Saranurak '20]
$\tilde{O}(n^{0.58})$	detection, det.	
$O(\Delta/\log n + \log\log \Delta)$	listing, det.	[Huang, Pettie, Zhang, Zhang '20]
$n^{1/3+o(1)}$	listing, det.	[C, Leitersdorf, Vulakh '22]

Triangles in Congest

Triangle detection?

Triangles in Congest

Triangle detection?

Cannot be solved in a single round:

- bandwidth $\geq \Delta \log n$ [Abboud, C, Khoury, Lenzen ‘17]
- bandwidth $\geq \Delta$ w. randomness [Fischer, Gonen, Kuhn, Oshman ‘18]

$\Omega(\log \log n)$ rounds [Assadi, Sundaresan ’25]

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Open problem:
Round complexity of **triangle detection** in Congest?

K_p in Congested Clique

$O(n^{1-2/p})$	listing	[Dolev, Lenzen, Peled '12]
$\tilde{\Omega}(n^{1-2/p})$	listing	[Fischer, Gonen, Kuhn, Oshman '18]
$\tilde{\Omega}(n/\log n)$	listing, BCC	[Drucker, Kuhn, Oshman '14]
$\tilde{\Theta}(m/n^{1+2/p})$	listing	[C, Le Gall, Leitersdorf '20]

Open problem:
Round complexity of K_p **detection** in Congested Clique?

K_p in Congest

$\tilde{\Omega}(n^{1-2/p})$	K_p listing	[Fischer, Gonen, Kuhn, Oshman '18]
$\tilde{O}(n^{5/6+o(1)})$	K_4	[Eden, Fiat, Fischer, Kuhn, Oshman '19]
$\tilde{O}(n^{21/22+o(1)})$	K_5	
$\tilde{O}(n^{3/4+o(1)})$	K_5	[C, Le Gall, Leitersdorf '20]
$\tilde{O}(n^{p/(p+2)+o(1)})$	$K_{p\neq 5}$	
$\tilde{O}(n^{1-2/p})$	K_p	[C, Chang, Le Gall, Leitersdorf '21]
$n^{1-2/p+o(1)}$	listing, det.	[C, Leitersdorf, Vulakh '22]

K_p in Congest

$\tilde{\Omega}(n^{1-2/p})$	K_p listing	[Fischer, Gonen, Kuhn, Oshman '18]
$\tilde{O}(n^{1-2/p})$	K_p listing	[C, Chang, Le Gall, Leitersdorf '21]
$\tilde{\Omega}(n^{1/2}/p)$	K_p Detection, $p \leq n^{1/2}$	[Czumaj, Konrad '18]
$\tilde{\Omega}(n/p)$	K_p Detection, $p > n^{1/2}$	

Open problem:
Round complexity of $\boldsymbol{K_p}$ **detection** in Congest?

C_p in Congested Clique

$\tilde{O}(n^{1-2/p})$	C_p listing	[Dolev, Lenzen, Peled ‘12]
Sparsity aware	C_p listing	[C, Fischer, Gonen, Le Gall, Leitersdorf, Oshman ‘20]

Open problem:
Round complexity of C_p **listing** in Congested Clique?

C_p in Congested Clique

$\tilde{O}(n^{1-2/p})$	C_p listing	[Dolev, Lenzen, Peled '12]
Sparsity aware	C_p listing	[C, Fischer, Gonen, Le Gall, Leitersdorf, Oshman '20]
$O(1)$	C_4 detection	[C, Kaski, Korhonen, Lenzen, Paz, Suomela '15]
$O(n^{0.158})$	C_p detection	
$O(1)$	C_p detection p is even	[C, Fischer, Gonen, Le Gall, Leitersdorf, Oshman '20]

Open problem:
Round complexity of C_p **detection** for even p in Congested Clique?

C_p in Congest

$\tilde{\Theta}(n^{1/2})$	C_4 detection	[Drucker, Kuhn, Oshman '14]
$\tilde{\Omega}(n)$	C_p, p is odd	
$\tilde{O}(n)$	C_p	[Korhonen, Rybicki '17]
$\tilde{\Omega}(n^{1/2})$	C_p, p is even	
$\tilde{O}(n^{1-1/p(p-1)})$	C_{2p}	[Fischer, Gonen, Kuhn, Oshman '18]
$\tilde{O}(n^{1-2/(p^2-p+2)})$	C_{2p}, p is odd	[Eden, Fiat, Fischer, Kuhn, Oshman '19]
$\tilde{O}(n^{1-2/(p^2-2p+4)})$	C_{2p}, p is even	
LB limitation	C_p, p is even	[C, Fischer, Gonen, Le Gall, Leitersdorf, Oshman '20]
$\tilde{O}(n^{1-1/p})$	C_{2p}, p=3,4,5	
$\tilde{O}(n^{1-1/p})$	C_{2p}	[Fraigniaud, Luce, Magniez, and Todinca '24 + '25]

Other Subgraphs

- $\forall p \geq 4$, $\exists H, |H| = p$, detection in $\widetilde{\Omega}\left(n^{2-\Theta(1/p)}\right)$ [Fischer, Gonen, Kuhn, Oshman '18]
- $\forall H$, detection in $\widetilde{O}\left(n^{2-2/(3p+1)+o(1)}\right)$ [Eden, Fiat, Fischer, Kuhn, Oshman '19]
- Induced subgraphs [Le Gall, Miyamoto '21, Nikabadi, Korhonen '21]

Dynamic

- Single-round bandwidth [Bonne, C '19]
- $O(1)$ amortized highly dynamic [C, Kolobov, Schwartzman '21]

Testing

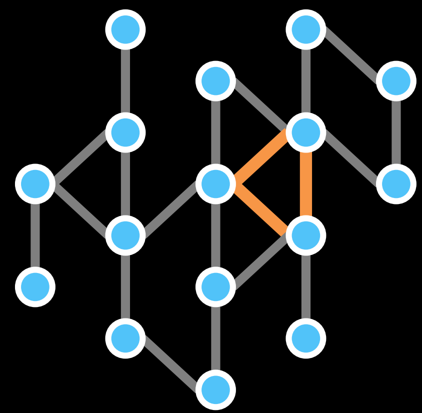
- Large cliques [Brakerski, Patt-Shamir '11]
  $(1/\epsilon^2)$ for triangles + more [C, Fischer, Schwartzman, Vasudev '16]
- $O(1/\epsilon)$ for triangles + more [Even, Fischer, Fraigniaud, Gonen, Levi, Medina, Montealegre, Olivetti, Oshman, Rapaport, Todinca '17]+[Fraigniaud, Olivetti '17]
- More [Fraigniaud, Rapaport, Salo, Todinca '16]

Quantum

- $\tilde{O}(n^{1/4})$ triangle detection [Izumi, Le Gall, Magniez '20]
- $\tilde{O}(n^{1/5})$ triangle detection + more [C, Fischer, Le Gall, Leitersdorf, Oshman '21]

Open problems

Triangle detection ?
K_p detection ?
C_p listing ?
C_p detection ?



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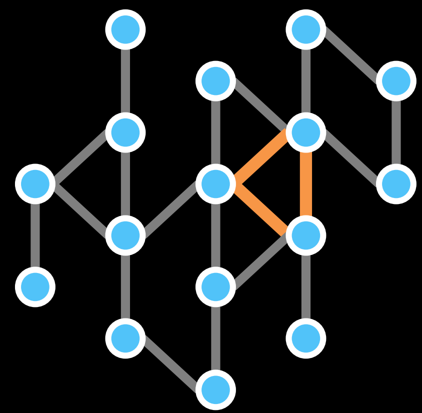

Questions?