Awake Efficient Distibuted Algorithms

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Durham University

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Today's Plan

- Introduction to the rest of the talk
  - Motivation
  - Model
  - Brief mention of the themes of the talk
- Leveraging sleep to gain something
- Dealing with global problems - enforcing structure on communication
- Leveraging structure on communication in local problems
- Final thoughts

Outline

Motivation

- Metric with increasing importance today - energy
- Multiple ways to approach this
  Simple approach - if a device not in use, turn it off temporarily
- Studied in wireless networks for multiple decades now [Nakano & Olariu, ICPP 2000]
- Recently started to be studied in wired networks [Chatterjee, Gmyr, & Pandurangan, PODC 2020]
- Since then, a plethora of work has come out in the wired setting [Chatterjee, Gmyr, & Pandurangan, PODC 2020, Barenboim & Maimon, DISC 2021, Ghaffari & Portmann, SPAA 2022, Dufoulon, Moses Jr., & Pandurangan, PODC 2023, Ghaffari & Portmann, PODC 2023, Augustine, Moses Jr., & Pandurangan, SIROCCO 2024, Ghaffari & Trygub, PODC 2024, Balliu, Fraigniaud, Olivetti, & Rabie, PODC 2025]

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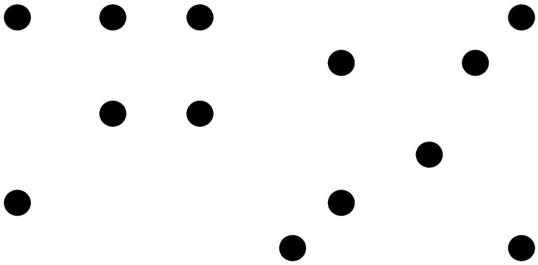
Distributed Computing Model



- n nodes with unique IDs $\in [1, N]$, $N = poly(n)$
- Arbitrary graph
- Global knowledge: n known to all nodes, sometimes N known to all nodes
- CONGEST model: $O(\log n)$ bits per message over an edge;
  LOCAL model: unlimited bandwidth

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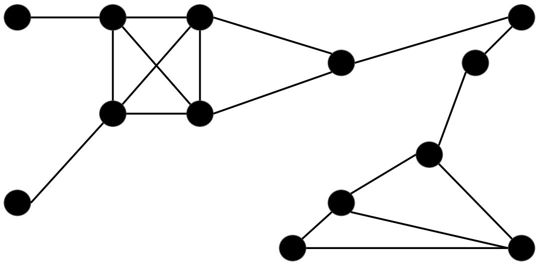

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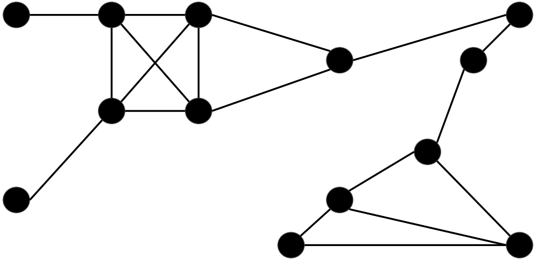

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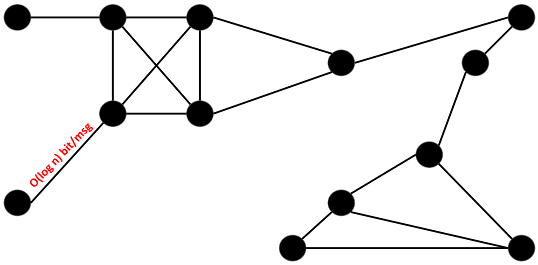

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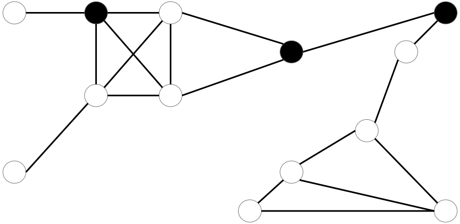
Awake/Energy Efficiency Defined & Measured



- Synchronous system
- In each round, each node's state $\in$ {awake, asleep}
- Algorithm designer controls when nodes are awake, asleep. Nodes know current round #
- Message exchange on an edge (u, v) only possible when u,v awake in same round
- **Metric 1:** awake time; **Metric 2:** running time/round complexity

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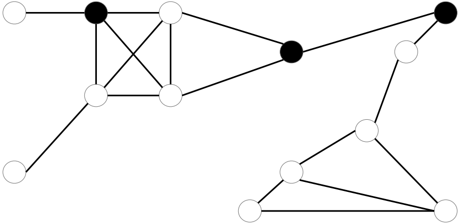

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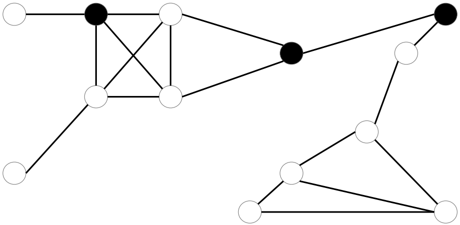

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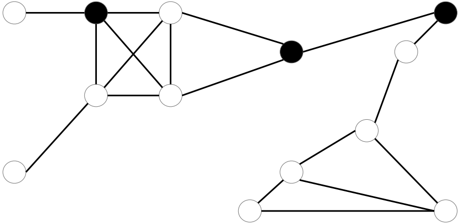

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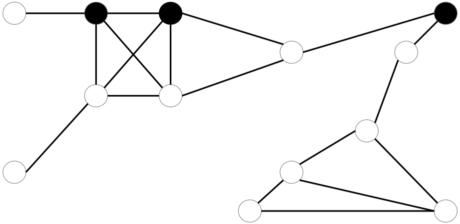

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General Themes of Talk

- Biggest algorithmic design hurdle: Figuring out when to communicate between neighbors
- Think of solutions as finding the communication structure inherent to pre-existing techniques, and then exploiting them
- It's ok to lose run time to enforce a structure which saves us awake time
- A little structure goes a long way, even for local problems

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Outline

Section Outline

Plan for Section

- Introduce solution to Maximal Independent Set (MIS) from [Chatterjee, Gmyr, & Pandurangan, PODC 2020]
- **Big Idea:** Cleverly putting nodes to sleep saves average node-awake time
- **Why care:** Lower bound [Kuhn, Moscibroda, & Wattenhofer, JACM 2016] - $\Omega(\min\{\frac{\log \Delta}{\log \log \Delta}, \sqrt{\frac{\log n}{\log \log n}}\})$.
  For high degree graphs, if nodes always awake, then total energy/awake time - $\omega(n)$
- [Chatterjee, Gmyr, & Pandurangan, PODC 2020]: $O(1)$ node-average awake time $\implies$ $O(n)$ total energy/awake time

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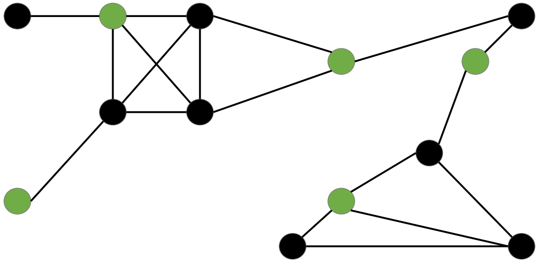
Maximal Independent Set (MIS) - Problem Definition

Maximal Independent Set (MIS)

For a graph $G(V, E)$, the maximal independent set of G is a set of nodes S such that for all nodes $u, v \in S$, the edge (u, v) does not exist and every node not in S has a neighbor in S.



Solution Outline

Each node samples $K = O(\log n)$ fair coin tosses

Each node participates in a recursive process MIS(K):

- Bottom out - either MIS(0) or you're awake & no one around you is in MIS (you're isolated): join MIS
- If undecided: if Kth coin toss heads participate in MIS($K - 1$), else sleep for duration of MIS(K-1)
- Inform neighbors of updates to your state (synchronize)
- If neighbor in MIS, don't join MIS
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Analysis

- Simple analysis - each node participates in $O(\log n)$ recursion, so each node $O(\log n)$ awake time
- **Insight:** At each level of the recursion, set of awake nodes A and sleeping nodes S. Nodes in set $MIS(A)$ have neighbors in set S.
  A constant fraction of S are pruned thanks to the actions of awake nodes
- In particular, at every level, $1/4$ of nodes in S pruned by nodes in $MIS(A)$. So at each level of recursion i, $(3/4)^i n$ nodes participate.
  These pruned nodes each need only $O(1)$ awake time for whole MIS process
- For non-pruned nodes, node average awake time $= (1/n)\sum_{i=0}^{K}(3/4)^i \cdot n = O(1)$

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Notes

- The given solution sacrificed run time to save awake time
  $O(\log^{3.41})$ run time
- However, it's possible to avoid doing so [Ghaffari & Portmann, SPAA 2022]
  $O(\log n)$ run time
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  Leveraged it to solve spanning tree.
  Also devised awake-efficient solution for class of problems called $O-LOCAL$
- **Focus of section** [Augustine, Moses Jr., & Pandurangan, SIROCCO 2024]: Tweaked structure so that minimum spanning tree could be solved. Additionally, run time improved
- **Big Idea:** Break up solution into smaller chunks of time where you can guarantee certain nodes are not awake at the same time
- **Why care:** Global problem, but $O(\log n)$ awake time. Also, the structure observed and built has applications to various other problems, both global and local

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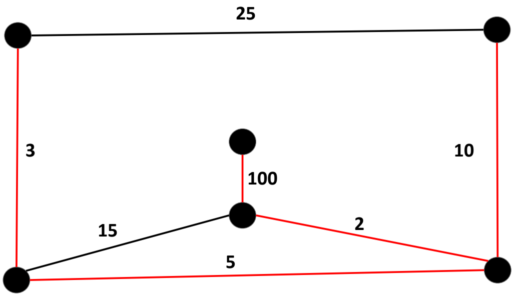
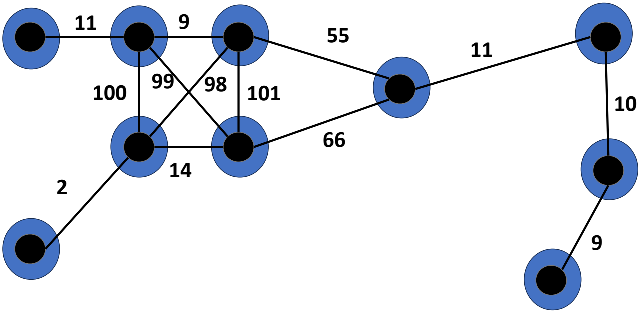
Minimum Spanning Tree (MST) - Problem Definition

Minimum Spanning Tree (MST)

For a graph $G(V,E)$ where each edge $e \in E$ has associated weight, the minimum spanning tree of G is a connected subgraph of G that connects all nodes of V, has no cycles, and has the minimum sum of edge weights amongst all possible subgraphs satisfying the previous two conditions.



Identifying Structure



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- Clusters (initially each node) merge using MOEs into larger clusters
- Each phase, constant fraction of clusters merge. Totally $O(\log n)$ phases

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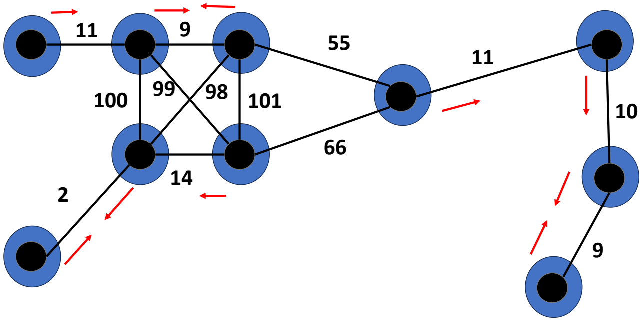

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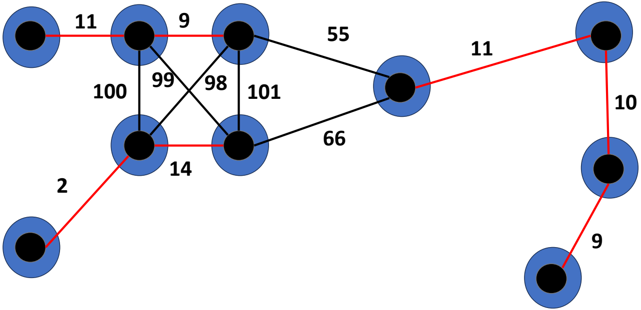

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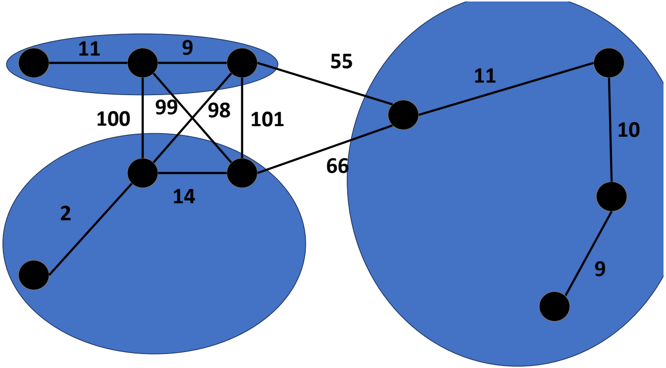

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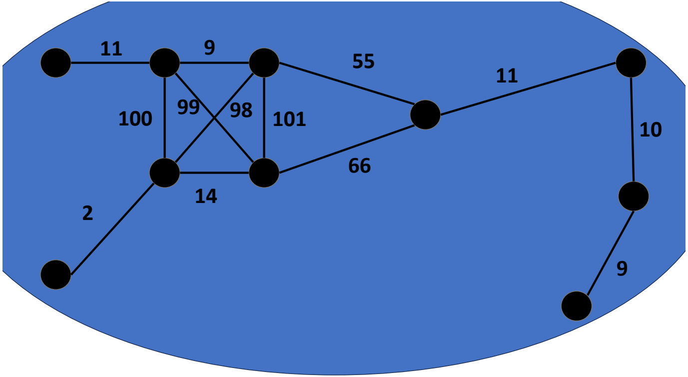

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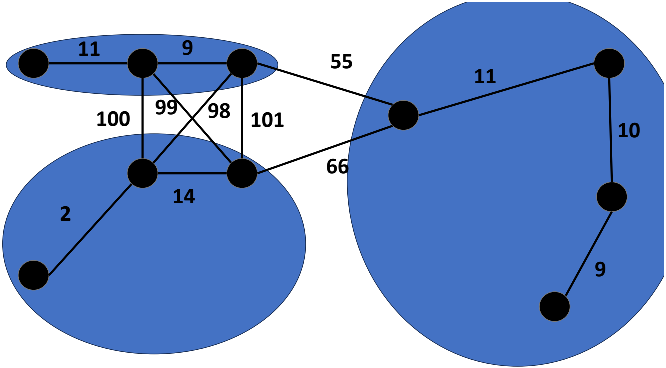

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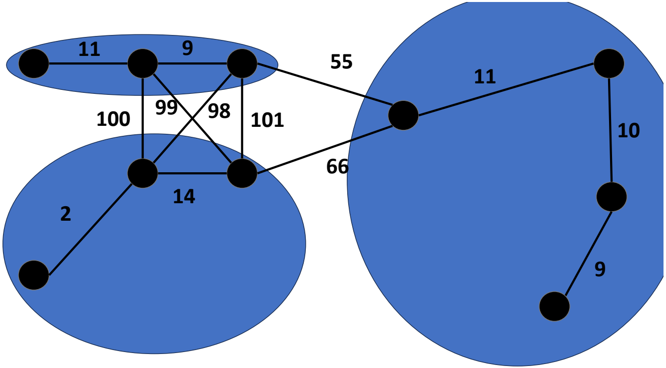

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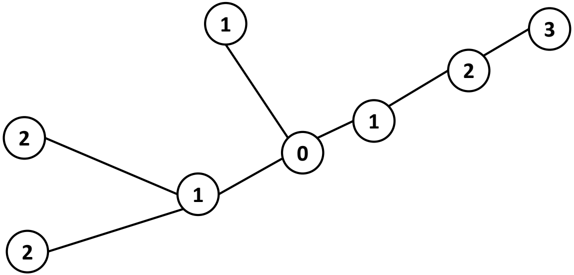

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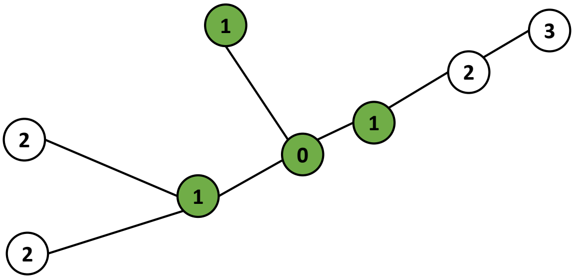

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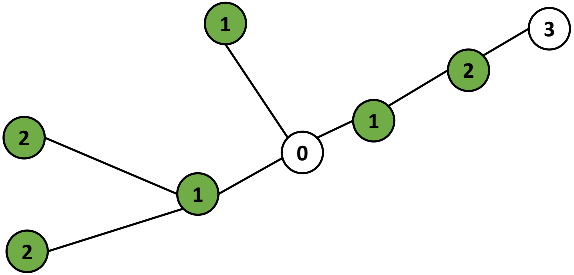

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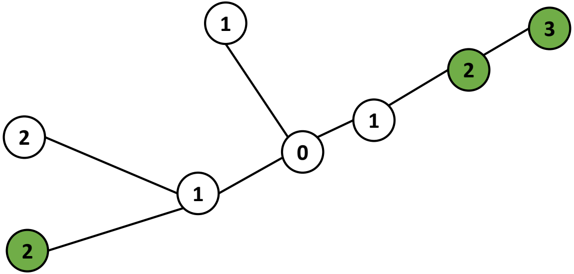

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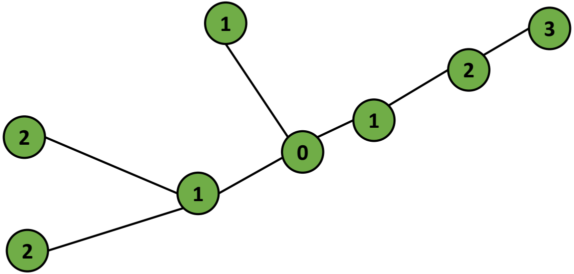

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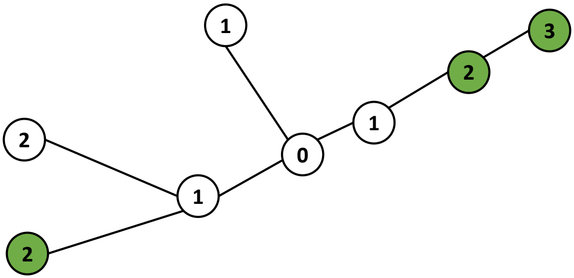

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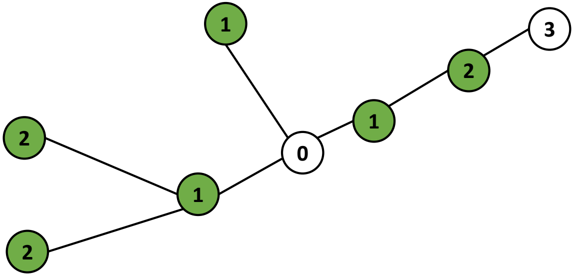

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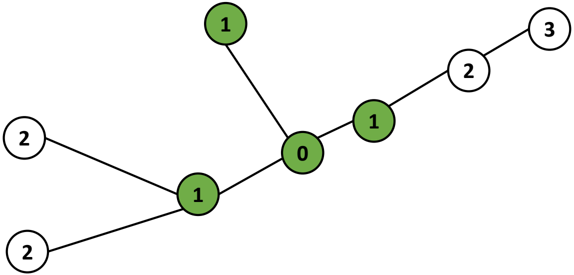

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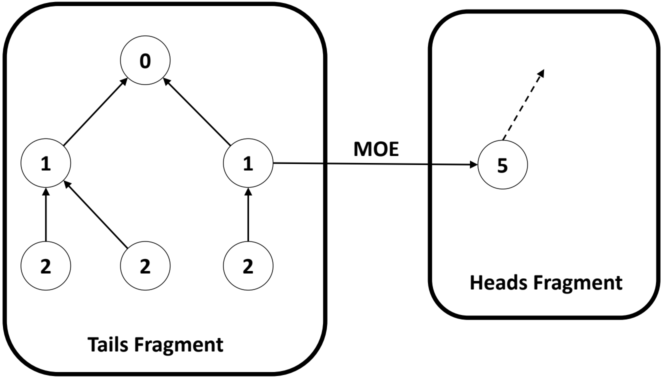

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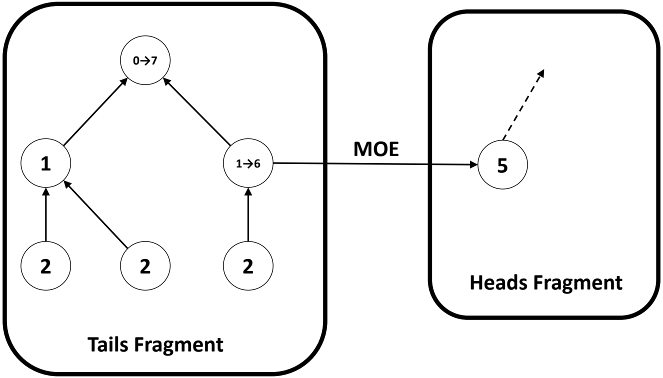

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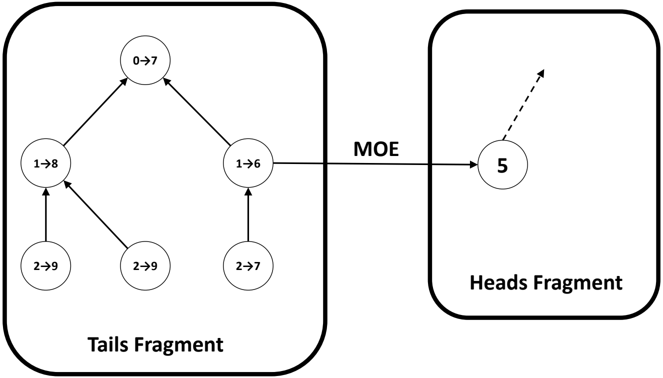

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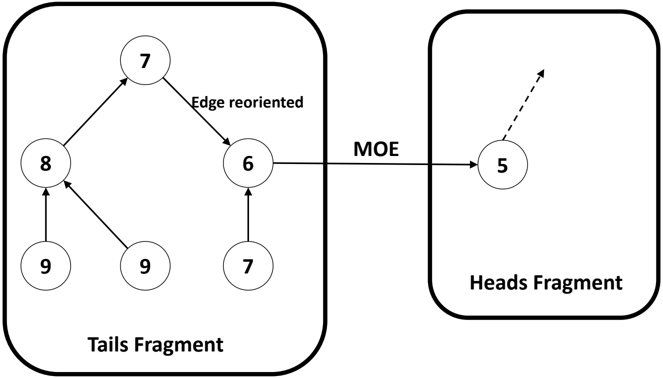

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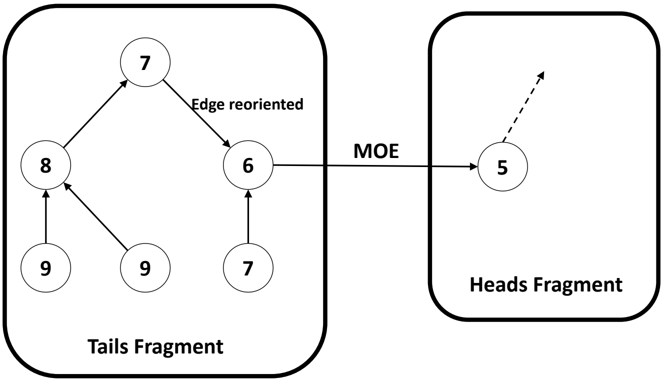

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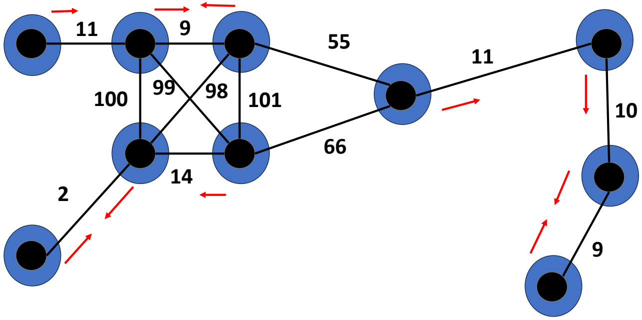

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- **Hint:** we want $O(1)$ awake time per node per phase.

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LDT of Size $O(\log n)$

- **Idea:** Shatter graph into disjoint non-neighboring sets of nodes
- Somehow ensure each set has at most $O(\log n)$ nodes
- Form an LDT on each set of nodes in $O(\log \log n)$ awake time per node
- Computing MIS on each LDT takes $O(\log \log n)$ awake time per node:
  (i) Generate IDs for all nodes and let them know in $O(\log \log n)$ awake time
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Konrad's Lemma

- **Idea:** Use a Lexicographically First MIS (LFMIS) (process nodes in inc. order of IDs)
- **Lemma:** Let t, t' be two integers such that $1 \leq t < t' \leq n$. Let V_t denote the (set of the) first t nodes, $V_{t'}$ the (set of the) first t' nodes and M_t the LFMIS over $G[V_t]$. Then, for any $\epsilon > 0$, $G[V_{t'} \setminus N(M_t)]$ has maximum degree at most $\frac{t'}{t}\ln(n/\epsilon)$ with probability at least $1 - \epsilon$
- Process first $O(\log n)$ nodes (based on ID) as a **batch**. Graphs have degree $O(\log n)$.
- Dividing nodes into $O(\log n)$ **buckets** u.a.r. guarantees any connected subgraph in each bucket has $O(\log n)$ nodes w.h.p.
- Run LDT on each of these connected subgraphs to get MIS, one bucket at a time
- Then once all buckets processed, process next batch of $O(\log^2 n)$ nodes. By Konrad's lemma, graphs have degree $O(\log n)$. Rinse & repeat
- Two issues (but really same issue): MIS from one bucket affects nodes in next. MIS from batch of smaller ID nodes affects MIS next one
- Essentially, how to communicate outcomes from smaller buckets/batch to larger buckets/batch?

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- Process first $O(\log n)$ nodes (based on ID) as a **batch**. Graphs have degree $O(\log n)$.
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- Then once all buckets processed, process next batch of $O(\log^2 n)$ nodes. By Konrad's lemma, graphs have degree $O(\log n)$. Rinse & repeat
- Two issues (but really same issue): MIS from one bucket affects nodes in next. MIS from batch of smaller ID nodes affects MIS next one
- Essentially, how to communicate outcomes from smaller buckets/batch to larger buckets/batch?

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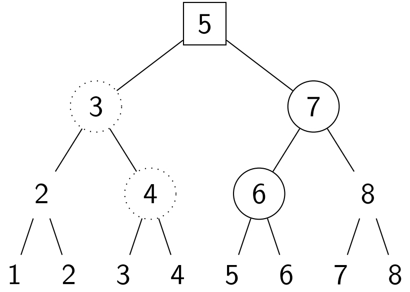

- **Idea:** Consider a binary tree where the ID of each bucket/batch is a leaf node. Systematic way to send messages from lower buckets to higher buckets in order. Ancestors of a leaf represent round numbers to be awake in a schedule the size of the tree
- **Important:** Ensure binary tree has only $O(\log n)$ nodes. Then height of tree (# of awake rounds per node) is $O(\log \log n)$

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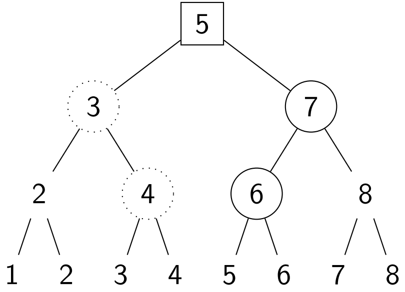

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Putting it all Together

- Start with the whole graph
- Nodes choose one of $O(\log n)$ batches u.a.r. - X random bits
- Nodes choose one of $O(\log n)$ buckets u.a.r. - Y random bits
- Nodes choose one of $O(\log n)$ IDs in LDT u.a.r. - Z random bits
- Each node has ID corresponding roughly to XYZ.
  Virtual binary tree takes $O(\log \log n)$ awake time to (i) solve LFMIS on LDT, (ii) communicate between buckets, (iii) communicate between batches
- LFMIS maintained. Totally $O(\log \log n)$ awake time per node

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Notes

- $O(\log \log n)$ awake time, $O((\log^7 n) \log \log n)$ run time.
  Variation: $O((\log \log n) \log^* n)$ awake time, $O((\log^3 n)(\log \log n) \log^* n)$ run time
- [Ghaffari & Portmann, PODC 2023] solve MIS in CONGEST - $O(\log \log n)$ awake time, $O(\log^2 n)$ run time.
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Outline

Final Thoughts

- Many more papers exist, not all covered in this talk
- Ideas from wireless models can be leveraged in wired settings
- Many problems still unstudied/scope for further study
- **Personal View:** The key to solving problems in this setting is to look at existing techniques for a given problem, identify who communicates with whom, and see if various techniques play well together on that front or if you can artificially make them play nice

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The End